ELEMENTARY
NOTIONS OF LOGIC

DESIGNED AS PROLEGOMENA TO THE

Study of Geometry

BY

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PREFACE

IT would be no recommendation, but quite the reverse, if a work of the kind that this professes to be were found to contain much that is new. Since, then, its matter can give it no claim to supplant other treatises, my little book, forced to plead for its life before judges at least impartial, possibly severe, must needs be forgiven a statement of its method and purpose somewhat more detailed than may at first sight seem modestly proportioned to its humble dimensions. It purports to be not so much a contribution to logical science itself as to the art of teaching it. I have long been driven to the conclusion that the rationale of the elementary teaching of a subject is in England too often fundamentally mistaken. Too often *Uebersicht* is put where *Einsicht* ought to be. Too often our elementary text-books resemble rather the lecture-notes of a clever student preparing for an examination than the thorough rounded work of the highly-trained teacher. The result is that the student finds himself bewildered by an immense amount of matter crammed into a small compass, and he must needs learn

his subject before he can understand his book. Now I venture to submit that all teaching should be *concentric;* by which I mean that a small area of the subject should be first mapped out and treated as completely as possible, then the boundaries of this area enlarged, and the new area as completely treated again; and so on, till the appointed task be done. At each enlargement of area, the advance into the wider range of thought should bring with it not only new notions, but that which is of infinitely greater importance—more enlarged views of the notions gained hitherto. The following pages contain an endeavour to treat the contents of the first and smallest area for the science of Logic, and in this, as in all other cases, the question must arise as to the boundaries of this first small area. In the present instance, however, the selection was ready to my hand; and I have written on so much of Logic as seems to me to be implied in, and necessary to, a just appreciation of the arguments of the First Book of Euclid. No one can be more painfully aware than myself of the many defects in the execution of this idea as now carried out. Nevertheless, my aim will be attained if, as I trust he may, the student, from carefully following the investigations of this little volume, gains a power which, aided by due industry and perseverance, shall enable him to read and assimilate the works of the great logicians. At the same time, I would also venture to hope that a reader who cannot proceed beyond the limits of one small volume may here obtain a knowledge sound and competent as far as it goes.

It will be as well to state here the exact relation of this work to geometrical knowledge. All the earlier portion

of the book can be read without any knowledge of geometry, but the geometrical examples should not be taken up until they can be treated concurrently with the propositions in Euclid to which they apply. When I speak of propositions in Euclid, however, I must not be understood to be declaring for the teaching of Euclid as distinguished from what is known as Modern Geometry, to which all my tastes and habits as a teacher strongly incline. And in order to make clear that the book applies as much to one as to the other, I have referred to Mr Wilson's Geometry throughout concurrently with Euclid, whenever geometrical examples are quoted. Logicians will not fail to recognize that the word "Prolegomena" on the Title-Page is used in a sense similar to that in which it occurs in Mansel's "Prolegomena Logica."

Some portions of the work have presented to me great difficulties. The chapter on Definition was the chief of these; and here I have largely departed from the treatment of that subject by Mill, and have more closely followed Archbishop Thomson—Mill's search for the ideally perfect definition being altogether too difficult for the beginner. But throughout the volume, in every case where I have been forced to adopt one view out of many, all supported by equally high authority, I have always been careful to give fair warning that different opinions will be met with in the course of a wider study of Logic. One or two minor points I have, to the best of my belief, worked out anew.

Though the book has been kept as elementary as possible, it has not been thought well to deny one or two glimpses into higher regions. Such, for instance, is the

identification of the *Reductio ad absurdum* with the ultimate process of human thought. This idea, in a somewhat different form, has recently received able statement from Professor Jevons, and is one which the late James Hinton, who also worked it out, was fond of saying that he had first learnt from the lines of Shelley's *Prometheus*—

> "In heaven-defying minds
> As thought by thought is piled, till some great truth
> Is loosened, and the nations echo round,
> Shaken to their roots."

It only remains to acknowledge my obligations to other writers. The admirable tractate "First Notions of Logic," by Professor De Morgan, intended by its illustrious author as an introduction to geometrical reasoning, is now out of print, and very rare. Had it not been so, the following pages would not have been written. But De Morgan's educational works, perfect as they are in conception and finish, are also intensely difficult—admirable for the teacher who has the courage to encounter them, but to the average pupil all but unapproachable. Hence a new edition of the "First Notions," which I had at first thought of undertaking, would not, as I soon found, have suited the purpose I had in view, even if I had been able in every instance to endorse the matter and approve the manner, which I could not always do. But I have freely consulted that work throughout the course of my own. I am also much indebted to the "Deductive Logic" of Professor Fowler, the "Principles of Science" of Professor Jevons, and the "Laws of

Thought" of Archbishop Thomson. The view I have adopted of the Import of Propositions is that which I learnt as a student from Professor Martineau, to whose instruction I shall ever look back as to the highest intellectual privilege it has been my lot to share. Special obligations, in addition to these general ones, I have acknowledged in foot-notes as they occurred.

The communication to me, under cover to the publishers, of any suggestions, and in particular any notification of difficulties that may be met with either by teachers or students, will be esteemed a favour.

A. M.

LONDON, *February 12, 1880*

CONTENTS.

LOGIC

CHAPTER I.

INTRODUCTORY ANALYSIS.

AS we go about our daily business, we are easily able to notice that the circumstances which surround us do not all affect us in the same manner. One event will produce in us a state of mind which differs from that produced by another event, not merely in degree, but also in kind. The fruit on the trees we put into our mouths, and *taste;* the birds sing to us, and we *hear;* one man loses a coin, and is *sorry;* another finds it, and is *glad;* a letter comes from a friend, and we *resolve* to go and see him; some difficulty meets us to be overcome, and we have to stop and *think.* In this way we discover our minds to be capable of four things, which are called by the names Sensation, Emotion, Volition, Thought. When we smell, taste, touch, hear, or see, we have Sensation. We may be glad, or sorry, or angry; we may love, or hate, or hope, or fear; these and many other states of mind are called Emotions. Volition occurs whenever we resolve, or make up our minds to do anything. The functions of Thought in general are too many and too complex to be treated here, but among the

chief of them is the one with which we shall be concerned, namely, *Reasoning.*

Perhaps the most common of all questions is contained in the single syllable "Why?" The little child who has but few words with which to express his thought, constantly shows his desire for that new knowledge, which to him is ever-growing power, by this question "Why?" And the learned man of science, who has grown old in the study of Nature, has all his life been working at questions of just this same form, "Why?" We all know how the answer to this question begins; every *Why* is followed by a *Because.* And whenever we allow ourselves to think that something has happened or will happen *because* of the happening of something else, we are said to *reason.* We reason whenever we believe one thing because we believe another. *Because* we see a red sunset this evening we believe it will be a fine day to-morrow: this is reasoning. I have never seen any swans but white ones; yet I believe there are some swans that are black, *because* my friend, who has been in Australia, tells me he has seen them, and I know my friend is a truthful man. This is a rather more complex example of reasoning. Or, again, if I say that all thrushes have a sweet note, and that bird in my garden is a thrush, *therefore* that bird can sing sweetly; here again I reason. From these examples we may see that reasoning takes place whenever, in ordinary life, we use the very common word *because,* or the less common word *therefore.*

Practically, the object of reasoning is to arrive at what is true. But in this book we shall only examine

the *process* by which we pass from one belief or one statement to another. We shall not concern ourselves in the slightest degree with the truth or falsity of either the statements from which we start or of those at which we arrive; all we shall have to attend to is how the process shall be rightly carried on.

Now, reasoning may be well done or ill done, but whether we reason well or ill we cannot avoid reasoning of some sort during every moment of our waking lives. But always remembering that reasoning is simply the process of passing from one belief to another, from one statement to another, we shall find that very good reasoning may land us in false beliefs, and bad reasoning in true ones. For if we start from what is false, and reason badly, we may arrive at something which is true; whilst even if we start from things that are false, and reason well, we may possibly arrive at what is true. But if what we start from is true, and our reasoning good, we can never arrive at what is false. For examples, we may examine one or two pieces of reasoning, or *arguments*, as they are called.

I. All bipeds are birds,
All men are bipeds,
∴ All men are birds.

Here the reasoning process is perfectly correct, and yet it is false that men are birds. The mistake must then lie in the statements from which we started, and we soon see that it is not true that *all* bipeds are birds, for there are other bipeds besides which are not birds, namely, men. We must, therefore, be careful not to say that the *reasoning* is bad because it

ends in what is false; for the reasoning may be quite good, and the mistake may be that we started wrongly. The journey has been safely performed, only we have got into the wrong train.

Nor must we think that the reasoning is necessarily good because we find it end in what we know to be true. Take, for instance—

II. Some black animals can swim,
Some men are black animals,
∴ Some men can swim.

Here the statements from which we started are both true, and the final statement we derived from them is true likewise; and yet a little careful attention will show that there is no justification for it contained in any part of the argument. For, perhaps, there are some other black animals which cannot swim, and the part of men which are black may, for aught we have said to the contrary, be just the portion of black animals that cannot swim. And, again, though some men are not black animals, we do not, so far as our argument is concerned, know anything about the capability to swim of any animals but black ones. Hence, we see that our knowledge of the fact that some men can swim must have been arrived at in some other way.

Before examining other cases, it will be desirable to explain more precisely what an argument is, and the parts of which it consists. To do this we will take an example of a perfect argument; for instance:—

(1) All birds are bipeds,
(2) All thrushes are birds,
(3) ∴ All thrushes are bipeds.

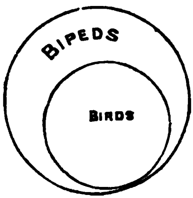
There are here three statements, which we have numbered (1), (2), and (3). Every complete statement of any kind of fact is called a **Proposition.** Of the three propositions in our argument, the first two are given us to start with, and these are called **Premisses.** We have nothing to do here with the truth or falsity of these two propositions; all we have to do is to see what legitimately follows from them. Combining them together by process of reasoning, we elicit from them a third proposition, and this third is called the **Conclusion.** As in the case of the premisses, it does not in any way matter to our present purpose whether the conclusion is such as we know to be true, or such as we know to be false. All we have to concern ourselves about is that the conclusion shall be just as true as the two premisses combined; that is to say, that, granting the truth of both the premisses, the truth of the conclusion must follow. Thus we have our argument analysed as follows:—

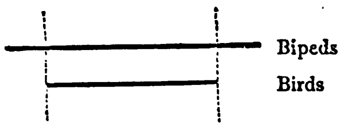
(1)		All birds are bipeds	Premisses.
(2)		All thrushes are birds	
(3)	∴	All thrushes are bipeds	Conclusion.

The premisses and the conclusion together constitute the whole argument, and an argument of this kind is called in Logic a **Syllogism,** from two Greek words, which mean "a putting together in thought." Thus a Syllogism is a combination in our thought of two propositions called premisses, from which combination there necessarily follows a third proposition, called the conclusion.

The above analysis by no means finishes our examination of the parts into which an argument or syllogism must be resolved. But it will be well, before concluding that analysis, to illustrate the meaning of the syllogism as a whole by reference to the following figures. We may show the exact meaning of the premisses, and their relation to each other, one by one, thus:—

(1) All birds are bipeds.



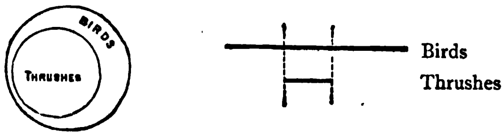


This proposition means that the whole class of objects called *birds* is included within, or contained under, the class *bipeds*. We may express this by a figure in many ways, of which the two most convenient are annexed. The class *bipeds* may be represented either by a large circle containing within it the smaller circle, which represents the class *birds*, or by a longer straight line, representing the class bipeds, and containing under it the shorter line which represents the class birds.

(2) All thrushes are birds.

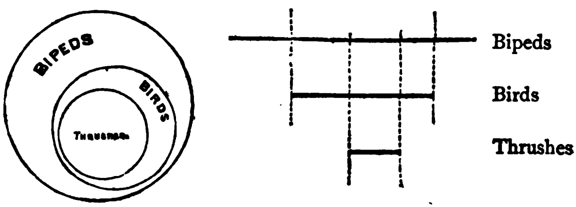
This other premiss can be represented in precisely

the same way, the larger circle, *birds*, containing within it the smaller circle, *thrushes*, or the longer straight



line, *birds*, containing under it the shorter straight line *thrushes*.

We are now in a position to combine these two figures, and thus to illustrate to the eye the process of thought of which the syllogism consists.



Here it is evident, from either figure, that if *thrushes* are contained within *birds*, and *birds* within *bipeds*, therefore *thrushes* are contained within bipeds.

The student is recommended to pay particular attention to these Methods of Notation, as they are called, particularly to that with the straight lines, which has over the method of circles many advantages that cannot be explained at this early stage. We shall soon, however, be able to show how to use these methods as *tests* of reasoning, and it will then be

seen how powerful they are as an instrument for detecting a mistake in the logical processes. The vertical dotted lines may be dispensed with in the notation with straight lines as soon as a little practice has rendered it familiar.

We have, then, so far, resolved our syllogism into three main parts, the two Premisses and the one Conclusion. Each of these three parts was found to be a Proposition, or statement; and we must now go on to find what is the construction of the Proposition.

Every time we make a statement of any sort, that statement must evidently contain the two elements—

(1) What we talk about;
(2) What we say about it.

Thus, if I say, "The dog barks," we have clearly here something I talk about, viz., *the dog;* and something I say about the dog, namely, that he *barks.* Now, we all know that what we talk about is said to be the *Subject* of our discourse. When a man writes a book, that book is written about something, and that something is the *subject* of the book. In just the same way, every proposition is a proposition about something, and that something is said to be the **Subject** of the Proposition. Thus, in the example given, the *dog* is the subject of the proposition, "The dog barks."

A very elementary acquaintance with grammar will have informed the student that the second element of the proposition, namely, what we say about the subject, is called the **Predicate.** Thus, *grammatically,* in our example, "The dog barks," *barks* is said to be

the predicate. We shall here, however, find a difference in the usage most convenient for the two sciences, Logic and Grammar, and another example will better display the *logical* meaning of the predicate. Take, for instance,

Man is mortal.

Here "Man" is the *Subject*, and "Mortal" is in logic called the *Predicate* (*mortal* being said to be predicated of *man*); while there yet remains the little word "is," which serves to connect the Subject and Predicate, and show that there is some kind of relation between them. This little word is called the **Copula,** and is always, when fully expressed, a part of the verb *To Be*, generally taking one of the forms *Is*, *Are*, with their opposites *Is not*, *Are not*.

The example, "Man is mortal," has always been a favourite one with logicians on account of its extreme simplicity of form. But the student must not expect to find all, or even many, propositions so simple in their structure as this one. Both subject and predicate may be expanded into very elaborate forms, as one or two examples will show, and as, in fact, has been already learnt from the study of grammar. Thus, if we say, "The habit of taking a cold bath every morning is conducive to health, happiness, and longevity," we have here a proposition in which both the subject and predicate are complex. What we talk about is "The habit of taking a cold bath every morning;" this, then, is the *subject*. What we have to say about that subject, or what we, in logical language, *predicate* of the subject, is "Conducive to health and

longevity;" whilst in this case the copula "Is" retains its explicit form, and is used, as always, to show a relation between the subject and the predicate.

The copula, however, will not always be expressed separately. In our very first example of a proposition, "The dog barks" (page 8), we shall not find any expressed copula, the copula being, in fact, involved in the predicate. Now, we may say either, "The dog barks" or "The dog is barking," and the student must remember that the latter form is the only one in which the proposition is capable of strictly logical treatment; and before proceeding to apply to any proposition those logical rules which we shall hereafter investigate, it will always be necessary to reduce it to the full logical form in which subject, predicate, and copula are all distinctly expressed. The transformation will always be easy when once the subject and predicate are clearly known. Thus—

Reading pleases me,

may be at once, and with no difficulty, altered to the logical form,

Reading is a-thing-which-pleases-me,

where the three elements of the proposition are made distinct.

The subject and predicate of every proposition are called its **Terms.** This word is derived from the Latin *Terminus*, a boundary, and means simply the two *ends* of the proposition, or rather the beginning and the end. Thus, in the example, "Man is mortal," *man* and *mortal* are the two *Terms*, which are connected by the copula *Is*.

This word "*Term*" is of great importance in logic, and the student is advised to pay particular attention to its exact meaning. In every proposition there must always be two terms, and never more than two. Taking the copula as the point of division, all that precedes it is one term, and all that follows it is the other term. Thus terms, like subjects and predicates (*supra*, pp. 8 and 9), may have any amount of complexity.

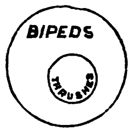
If we now return to our syllogism, we shall find ourselves able to complete its analysis.

(1) All birds are bipeds,
(2) All thrushes are birds,
(3) All thrushes are bipeds.

We have already found that this syllogism (and every other) consists of three propositions, the first two being given us, called Premisses, and the third evolved from these premisses, and called the Conclusion. Further examination will show that there are in the whole syllogism only *three* terms, viz., *Birds*, *Bipeds*, *Thrushes*. This is worthy of note, because since there are three propositions, and each proposition must have two terms, a syllogism which contains three propositions might be supposed to be capable of containing six terms. But no perfect syllogism can contain more than three terms—a law the reason of which will soon be apparent.

Taking the conclusion, "All thrushes are bipeds," we find that the relation between its two terms is a relation of containing and contained; the objects which make up the class *thrushes* being all contained in the

class *bipeds*, a relation which we have found (*supra*, p. 6) we can express to the eye thus:—



Now the term *bipeds*, which is expressed by the larger containing circle, which term is also the *predicate of the conclusion*, is called the "**Major Term**," and the term *thrushes*, expressed by the smaller circle, which is also the *subject of the conclusion*, is called the "**Minor Term.**" Looking once more at our syllogism, we find that each of these terms occurs again. The major term *bipeds* occurs in (1), where it is asserted in effect that the class *bipeds* contains the class *birds*. The minor term *thrushes* occurs in (2), where in like manner it is asserted that the class *thrushes* is contained in the class *birds*. We thus find that what the premisses really give us is the relations between the major and minor terms respectively, and another term, *birds*. This other term, with which both the minor and major terms are brought into relation by the premisses, is called the "**Middle Term.**" The major and minor terms, being brought into relation by the premisses with the same middle term, are really thus brought into a relation with each other, explicitly stated in the conclusion, and in accordance with the following law:—

"Whatever can be predicated of a class can be predicated of each and all of the individuals contained in that class."

This law is of the very highest importance, and is known as the **Dictum de Omni et Nullo** of Aristotle, a Latin phrase meaning "A saying about all and none."

CHAPTER II.

OF TERMS.

Definition.—A word, or assemblage of words, capable of standing by itself as the subject or predicate of a proposition is called a **Term.**

It will be well to compare this definition with certain facts which have previously been learnt from grammar.

A very little examination will be enough to show that the only *single* words capable of being terms are Nouns, Pronouns, Adjectives, and Participles. These are, in fact, the only words which can have a complete meaning by themselves, independently of the aid of other words. Thus in "Birds are bipeds," both subject and predicate are nouns; "Coal is black" is a proposition formed with a noun and an adjective; "I am hot" contains a pronoun and an adjective. Verbs are not generally held to be terms, because they are terms and something more, namely, the copula. Thus, in "The dog barks," the verb "barks" includes the predicate and the copula, the whole proposition being

equivalent to the logical form, "The dog is barking." Thus, for logical purposes, we have the equation

Verb = Copula + Participle.

Phrases may run to any length, and still only constitute a single term. This will be easily remembered if it be clearly grasped that every proposition must have two, and can only have two, terms. Thus, in the proposition, "To try to find a guide of life superior to reason, is to attempt to soar above the atmosphere on wings," we have two long terms connected by the explicit copula.

CHAPTER III.

OF THE DENOTATION AND CONNOTATION OF TERMS.

The various ways in which terms can be divided and classified do not fall within the limits of this work. There is, however, one distinction with regard to terms that is too important to be omitted.

Taking, for example, any common noun, such as *Dog*, we find that it calls to our minds two quite different sets of ideas, one consisting of the *objects* to which the name applies, and the other of the *qualities* which those objects possess, and in virtue of which they receive the name. Thus Dog is the name of each and all of a particular kind of animal; and the word *Dog* also implies a particular form, colour, size, etc., all varying between certain limits only. Now the sum total of the objects to which the term *applies* is

its **Denotation,** while the sum total of the qualities or attributes which the term *implies* is its **Connotation.** Proper names have, strictly speaking, no connotation; they are only the marks, as it were, which we set upon things to know them again; and some terms, including, for instance, all adjectives, have only connotation, and do not serve to designate any particular objects.

There is a peculiar law governing the relation between the denotation and the connotation of terms, which is worthy of very careful attention. It is a law which has been the subject of much learned writing, and which has been very variously stated in the many attempts that have been made to give it accurate expression. In its most general form it may be stated thus:—

The larger the Denotation of a Term, the smaller its Connotation; and conversely, the larger the Connotation, the smaller the Denotation.

It must be distinctly understood that the wording of the above law must be taken in a rough general sense, and not as implying any formula of mathematical accuracy. It does not mean that if you double the denotation you halve the connotation; thus making them, in mathematical language, to "vary inversely." It must only be taken to imply that a larger denotation goes with a smaller connotation, and that of two terms, A and B, if the denotation of A is larger than the denotation of B, then the connotation of A is smaller than the connotation of B; and by adding one more to the qualities to be connoted, we may decrease to an indefinite extent the

number of objects denoted. This rule we will proceed to exemplify.

Suppose we take the two terms *Flower* and *Rose*. Now it is clear that the word *Flower* may be applied to every object to which we can apply the word *Rose*, and to many more. That is to say, the denotation of the word *Flower* is larger than that of *Rose*. On the other hand, it is clear that *Rose* expresses as many qualities or attributes as *Flower*, and some more; that is to say, all the attributes which are common to every flower, together with certain others which are peculiar to roses. *Every* flower must have certain qualities, and all these are sure to be found in the rose, as in the buttercup, and in all other flowers. But, besides these, the rose has certain attributes of its own, its peculiar form, fragrance, etc., which the buttercup does not possess, and which are found in no other flowers but roses. Therefore, the term *Flower* has a larger denotation than the term *Rose*, inasmuch as it applies to more numerous objects; while *Rose* has a larger connotation than *Flower*, inasmuch as it implies more numerous attributes. And again, if to the qualities already connoted by *Rose*, we add one more, *White*, the denotation will be indefinitely diminished, for every other shade of Rose will now lie outside the denotation of the term *White Rose*.

A good instance of this law may be found in Arithmetic. An intelligent study of the elements of integral arithmetic will have very early shown that quantities can only be added and subtracted where they are of the same kind. Thus, we cannot find the sum of

$$5 \text{ horses} + 3 \text{ dogs},$$

because the denotation of both *Horses* and *Dogs* is too small for either to include the other. Logically, however, a solution can be found. We may abandon that portion of the connotation of each term which is peculiar to it, and by that means, since to lessen the connotation is to increase the denotation, we may, as it were, find a sort of Logical Common Multiple of the two Denotations. Thus *Horse* implies all the qualities of *Quadruped* and some more; *Dog* implies, in like manner, all the qualities of *Quadruped* and some more. Dropping out then the "some more" in each term, we find the common connotation for each "having four legs,"* which accompanies a denotation large enough to take in both, and we obtain—

5 horses + 3 dogs = 8 quadrupeds.

In this case the student will see that the smallest class has been selected which is capable of including both horses and dogs. We might have selected a larger class, "animals," and then we should have had

5 horses + 3 dogs = 8 animals.

In this case, however, we should have surrendered more than was necessary of the connotation of each of the addenda, for both *horse* and *dog* imply the quality of having four legs, whilst *animal* does not imply any particular number of legs, or any legs at all. Thus both *quadruped* and *animal* are logical common multiples of the denotations of *horses* and *dogs*, but

* Strictly speaking, to this must be added all those qualities which the zoologist would know to be implied in the term "Quadruped."

the term *quadrupeds*, where we have made the smallest possible sacrifice of connotation consistent with obtaining the required amount of denotation, may be considered the Logical Least Common Multiple of the two denotations.

CHAPTER IV.

OF GENUS AND SPECIES.*

Definition.—When a larger class contains under it a smaller class (as *Birds* contains *Thrushes*, etc.), the larger class is called, with respect to the smaller class, a **Genus**, and the smaller, with respect to the larger, is called a **Species.**

It is possible to arrange a succession of smaller and smaller classes, of which each larger one shall contain all the smaller ones within it, like the Chinese boxes which fit one within another. Thus, there is a certain kind of objects which we call *Animals.* Some of these animals have a backbone, and these animals we call *Vertebrata.* Some of these vertebrata are found to have feathers and wings by means of which they can

* The sense in which these words are here used and explained is that in which they are most commonly and popularly used in the ordinary language of life. The student who intends to pursue the study of logic beyond the limits of this work will do well to take an early opportunity of consulting on this point Prof. Jevons's "Principles of Science," 3d Edition, p. 698, and Mill's "Logic," Bk. I., Chap. vii., § 3.

fly in the air, and for these and other reasons we call them *Birds*. And one particular kind of bird is called by the name *Thrush*. In this way, it is clear, we can get a succession of classes — Animals, Vertebrata, Birds, Thrushes. Of these the largest is always a genus and the smallest always a species, whilst the intermediate classes are species with respect to the larger classes in which they are contained, and genus with respect to the smaller classes they contain. Thus in the succession Animal, Vertebrate, Bird, Thrush, we find that *Animal* contains within it all the others; *Vertebrate* contains *Bird* and *Thrush*, and is contained in *Animal; Bird* contains *Thrush*, and is contained in *Animal* and *Vertebrate; Thrush* is contained in all the others. *Animal*, therefore, is a *genus* with respect to all the others; *Vertebrate* is a species of *Animal*, and a genus with respect to *Bird* and *Thrush; Bird* is a species with respect to *Animal* or *Vertebrate*, and a genus with respect to *Thrush; Thrush* is a species with respect to any and all of the others.

Now let us examine in what way the genus differs from the species immediately below it. Taking the first in our succession of classes, we shall find that the word "Animal" can be applied to many objects which are not "Vertebrata," but that "Vertebrate" cannot be applied to any object to which "Animal" cannot also be applied. In other words (See Chap. iii.), the denotation of "Animal" is larger than the denotation of "Vertebrata." But of two terms, if the denotation of the first is greater than the denotation of the second, the connotation of the first is smaller than the connotation of the second (p. 15). Hence we shall

expect to find that the connotation of the term expressing the genus is smaller than that of the term expressing the species, and this we can see to be the case in our example. The word "Vertebrate" expresses all the attributes expressed by the word "Animal," and in addition it points to the possession, by the objects it designates, of a particular form of bone. Now this particular quality or attribute, by the possession of which the species is marked off from the rest of the genus in which it is contained, is called in Latin the *Differentia*, or in English the *Specific Difference.* And the connotation of the word expressing the genus, together with the connotation of the word expressing the difference, will always exactly make up the connotation of the word expressing the species; a law which is generally more shortly stated thus—

Genus + Differentia = Species.

That genus to which any species belongs, and which differs from that species by the smallest possible *differentia* is called the *Proximum Genus.** Thus in the series above given (p. 19) both "Animal" and "Vertebrate" are genus with respect to "Bird;" but "Vertebrate," which differs from "Bird" by the smallest possible difference of connotation, is the Proximum Genus. And the Proximum Genus of two different species, will be another name for what has been called above the logical least common multiple (p. 18). The true *proximum genus*, however, is often somewhat hard to discover. Thus the student will

* Latin, *Proximus*, nearest.

learn very early in geometry that the species *Circle* is contained within the genus *Plane Figure.* And this must serve for a *proximum genus* until, after considerable progress in the science, it gives place to *Conic Section*, the truly nearest genus in which *Circle* can be included.

CHAPTER V.

OF PROPOSITIONS.

WHENEVER we express our thoughts in words, it will be found on examination that such expression can only take two forms—we must *affirm* or *deny.* And every affirmation or denial is called a *statement*, or **Proposition.**

Thus, for instance, if I select some such quality as *blackness*, and some such object as *dog*, I can combine these in a Proposition in two ways, and only two—thus, "The dog is black," "The dog is not black."

It might seem at first sight that there are some propositions which neither affirm nor deny, but this cannot be the case. For suppose we have such a proposition as "I do not know whether the dog is black or not," it may be said that here I neither affirm nor deny the blackness of the dog. And that is certainly true; but then this proposition is not about the dog, but about *me*, and affirms that I do not know something. Its full logical form would be, "Whether the dog be black or not is a thing which I do not

know," and in this form an affirmation may be clearly seen to be made.

The above remarks will prepare the student for finding that it will sometimes be difficult to reduce a proposition from its colloquial to its logical form. A well-known example of this difficulty may be seen in the proposition, "None but the brave deserve the fair," which may be reduced to the logical form, "All who deserve the fair are brave." A little practice will, however, soon render the process of reducing propositions to their logical form familiar and easy.

It has already been pointed out what are the elements out of which a proposition is constructed (p. 8). We there found that we have in logic three elements in a proposition—the Subject, the Predicate, and the Copula. Of these the Subject and Predicate have been sufficiently explained above, but the Copula will require further examination.

The Copula will always be part of what is called in grammar the Substantive Verb, *i.e.*, the verb *To be*. It will always, when *explicit* (p. 10), take some one of the forms "Is," "Are," "Is not," "Are not," &c. Now there lies behind this simple and apparently innocent fact a great danger, the copula being in reality, by its very nature, a trap for careless reasoners, for the real function of the copula in logic is only to point out that an affirmation or denial has taken place, and *nothing more*. But the words "Is," "Are," "Is not," "Are not," have in common use another and very different meaning: they imply a real existence. And these two meanings must be carefully distinguished, as the neglect of such distinction has led to endless confusion

in every region of thought. Thus, suppose we say "A dog is a quadruped," we do not, logically speaking, make any assertion about the existence of the dog. For aught we have said to the contrary, there may be no such things in the world as dogs of any kind, quadrupeds or otherwise. If this assertion seems at first sight rather startling, we have only to look at another proposition of precisely the same logical construction, to find that it is nevertheless correct. Take, for example, "A dragon is a winged fire-breathing serpent." In logical structure the two propositions are exactly the same; and any one who inferred from the first the real existence of dogs, must be prepared to admit, on the evidence of the second, the real existence of dragons.

Thus it must never be forgotten that the whole function of the copula is to point to the fact that there is a relation, either of affirmation or of denial, between the subject and the predicate of the proposition.

The precise nature of the relation which is thus expressed in every proposition is a great matter of controversy amongst logicians, and the student who desires to get to the bottom of this question must consult more advanced works on logic, where, under the title "The Import of Propositions," he will meet with able arguments for very various views. The following is one view which the beginner will do well to accept, at least until he has had opportunity of comparing it with others, and forming an independent judgment.

Import of Propositions.—Taking any proposition, *e.g.*, "A dog is a quadruped," we shall find

that what we really say is that all the objects called *dogs* possess every quality, in virtue of which both they and many other objects receive the name *quadruped.* That is, we are really declaring in our proposition that the objects to which the subject *applies* possess the attributes which the predicate *implies;* or, in other words, we are uniting the connotation of the predicate to the denotation of the subject. And the business of the copula is simply to form a link between these two elements, and show that the union has been effected in thought.

Hence all propositions imply that the connotation of the predicate can be united in thought to the denotation of the subject.*

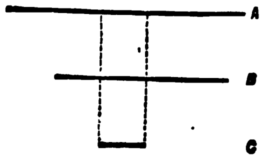
In considering the logical proposition we shall have no concern in this work with its truth or falsity. All we have to concern ourselves with is its form. This has been fully explained above in Chapter I.; but to draw more forcible attention to the fact that it is the *form* and not the *matter* of the proposition which we have to investigate, it has been the custom of logicians to substitute symbols for the significant terms of propositions, as thus:—

Take any Syllogism, or piece of reasoning—*e.g.*, the one we examined on page 5. We there found three terms—*Bipeds*, *Birds*, *Thrushes*—which we called respectively the Major, Middle, and Minor terms.

* For other views the advanced student should consult the chapter on the Import of Propositions in Mill's Logic.

Let the term *Bipeds* be represented by the letter A, *Birds* by B, and *Thrushes* by C. Then

For	Write
All Birds are Bipeds.	All B is A.
All Thrushes are Birds.	All C is B.
∴ All Thrushes are Bipeds.	∴ All C is A.



We can here see that the form of the argument is exactly the same when we reason with letters which may stand for anything, as when the argument is concerned about terms whose meaning we know. This is a point to which we shall have to return hereafter; but what has been said was necessary in order to point out forcibly to the student the fact that he will be concerned only with the relation of term to term, and not at all with the meaning of those terms.

CHAPTER VI.

OF QUALITY AND QUANTITY.

We have seen in the last chapter that every Proposition is either an affirmation or a negation. This will decide for us the division of Propositions according to their **Quality**. By long usage amongst logicians, Propositions conveying an affirmation are said to be

of *Affirmative Quality*, and Propositions conveying a negation are described as of *Negative Quality*.

There is another distinction of propositions which will require a little more care. Compare the two propositions,

All Dogs are Quadrupeds; Some Dogs are Black.

In the first of these examples the proposition affirms that the qualities implied in the word *Quadrupeds* are found in all and every of the objects to which the name *Dog* applies. In the second example, it is only affirmed that the quality *Black* belongs to a portion of the objects denoted by the name *Dog*. The first of these propositions is therefore said to be *Universal*, and the second *Particular*.

With regard to the Particular proposition, great care must be taken to distinguish its logical force from its colloquial use. Such a proposition as "Some dogs are black," is generally understood to mean that some dogs are, and some are not, black. But the student cannot be too often cautioned that for logical purposes he must concern himself only and solely with what is said in the propositions before him, and must not allow himself to supply anything at all from his own knowledge of the subject-matter of those propositions. Thus compare the propositions—

(*a*) Some triangles are three-sided;
(*b*) Some triangles are equal-sided.

The student will soon be able to perceive that in (*a*) "Some" might be changed to "All" without any departure from the facts of the case, since there are no triangles with any other number of sides. In (*b*)

however, this is not the case, there being many forms of triangles which are not equal-sided. Nevertheless, logically, and for the purposes of argument, these two propositions must be considered as exactly the same, since both can be translated into the same symbolic form,

Some A is B.

In this form we make an affirmation about some certain portion of A; but we are so far from affirming that another portion of A is not B, that we do not even affirm that there is any other portion of A existing at all. The particular proposition must therefore in logic be always understood to mean exactly what it says, *and nothing more.**

We have thus propositions divided into

(*a*) Affirmative and Negative;
(*b*) Universal and Particular.

These two divisions being independent of each other, may be combined. We thus get four kinds of propositions :—

Universal Affirmative, as, All S is P.	(A)
Universal Negative, as, No S is P.	(E)
Particular Affirmative, as, Some S is P.	(I)
Particular Negative, as, Some S is not P.	(O)

The vowels affixed in brackets to these propositions have been used throughout almost the entire history of the science to designate them, and to enable pro-

* For a more general explanation of this, see below, Chap. vii.

positions to be readily classed under one of the four heads above given. Thus, a universal affirmative is known as an "A Proposition," a particular negative as an "O Proposition," etc.; or even more shortly still, as an "A" or an "O," etc.

For a more extended division and classification of propositions, the advanced student may consult Sir William Hamilton's Lectures on Logic.

CHAPTER VII.

OF THE OPPOSITIONS AND CONVERSIONS OF PROPOSITIONS.

OPPOSITION.

Definition.—Two propositions are said to be **in opposition** when, having the same subject and predicate, they differ in quality or quantity, or both.

The Qualities of propositions are, as we have seen (pp. 26, 27), two—Affirmative and Negative. Now, the relation in which a negative proposition stands to the corresponding affirmative will be found to be simple and easily understood, though not quite the same in Logic as in ordinary conversation.

In the ordinary converse of life, the negation of one proposition generally points with more or less clearness to the affirmation of another with the same subject and predicate, and of the same quantity, but

differing from it in quality. We have already seen (p. 26) how the particular affirmative, "Some dogs are black," is generally held to imply the corresponding particular negative, "Some dogs are not black." By ordinary usage, a similar rule holds good to an even greater extent with the Universal proposition. Thus, to the child's question, "Is my sum right?" the answer, "No," certainly implies that the sum is wrong. Here the negative of a universal proposition implies its direct contrary, there being only, by the nature of the case, the two alternatives, "Right" and "Wrong." Even where there are several alternatives, the simple negation of one will often amount to something very like a direct affirmation of some other. Thus a person who asks, "Do I turn to the right for London Bridge?" and receives the answer, "No," will probably walk straight on, though he might, of course, either turn to the left, go back the way he came, or stand still. And where there are many intermediate degrees possible between the affirmative and its contrary, the negation of it will often affirm its contrary, and neglect the intermediate degrees. The simple answer "No" to the question, "Is it a long race?" generally means not only that it is *not above* the average length, but that it falls considerably *below* it, and that the race is, in fact, a short one. The extremely frequent occurrence in conversation of emphatic negative phrases ("Not at all," "By no means," etc.), and of small expletives ("Oh no," "Oh dear no," etc.), renders this use of the negative by far the most common in daily life.

But with this common usage we shall have here no

concern. Logically, we must never forget that we must fix our attention on what a proposition says, ***and nothing more.*** (Cf. p. 27.) "To turn to the right is not the way to London Bridge" is a proposition which must be considered to leave us perfectly in the dark as to whether London Bridge lies in front, behind, to the left of us, or under our feet. The statement, "It is not a long race," does not justify any belief that it is a short one; even "It is not right" does not *logically* declare that "It is wrong." All these pieces of information are obtained from our knowledge of the subject-matter about which we are reasoning; but the nature of the subject-matter has nothing to do with the nature of the reasoning.

The real understanding of negatives in logic is a point of considerable importance, and the student cannot do better than carefully to think his way through the following exposition of the subject by Professor De Morgan :—

"The negative words 'not,' 'no,' etc., have two kinds of meaning, which must be carefully distinguished. Sometimes they deny, and nothing more; sometimes they are used to affirm the direct contrary. In cases which offer but two alternatives, one of which is necessary, these amount to the same thing, since the denial of one and the affirmation of the other are obviously equivalent propositions. In many idioms of conversation the negative implies affirmation of the contrary in cases which offer not only alternatives, but degrees of alternatives. Thus, to the question, 'Is he tall?' the simple answer, 'No,' most frequently means that he is the contrary of tall, or considerably

under the average. But it must be remembered that in all logical reasoning the negation is simply negation, and nothing more, never implying affirmation of the contrary." *

Now, it is plain that if we take a Universal Affirmative, we may oppose to it either a universal negative or a particular negative. Thus we may oppose the A proposition, "All dogs are black," either by the E, "No dogs are black," or by the O, "Some dogs are not black." The opposition between an A and an O, where the one proposition denies *some portion* of what is affirmed by the other, is called **Contradictory Opposition.** The opposition between two Universals differing only in quality—*i.e.;* the opposition of two propositions, one of which denies *everything* that the other affirms—is called **Contrary Opposition.** A contrary, therefore, is a complete and total contradictory. We have, then—

To	(A) All S is P,
are opposed	(O) Some S is not P (*contradictory*)
	(E) No S is P (*contrary*)

These oppositions are subject to the two following very important laws :—

(i.) **Of two Contraries, both may be false, but both cannot be true.**

It is false that "All dogs are black;" it is also false that "No dogs are black." But if it be true that all men are mortal, it certainly cannot also be true that none are.

* *First Notions of Logic*, p. 5.

(ii.) **Of two Contradictories, one must be false and the other true.**

If it be false that "All dogs are black," then it is certainly true that, if such things as dogs can be found at all, some dogs can be found of some other colour—that is, that "Some dogs are not black."

Now, we saw that the E proposition is opposed to the A proposition in quality only, and the O proposition in both quality and quantity. There remains the I proposition, which is called the **Subaltern** of A, and which differs from it in quantity only. It is said to be "opposed" to the A merely for the sake of symmetry of language. That there is really no opposition the law of their relation will show, as follows:—

(iii.) **The truth of the Universal implies, but is not implied by, the truth of the Subaltern.**

If it be true that "All dogs are quadrupeds," it certainly follows that some are; but because "Some dogs are black," it does not follow that all are.

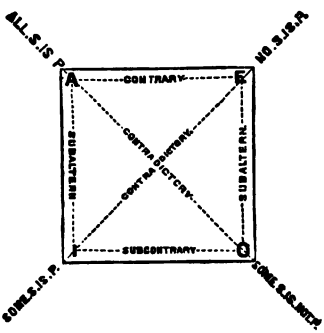
There now only remains to be considered the opposition of two particular propositions which differ in quality—the opposition of I and O—called **Sub-contrary Opposition.** Take, for example (I), "Some dogs are black," and (O) "Some dogs are not black." If we examine the class *dogs* to find out what truth there is in these propositions, it is clear that there are only three possible results of such examination, one of which must occur. We shall find that—

either	All dogs are black.	(*a*)
or that	No dogs are black.	(*b*)
or that	Some dogs are and some are not black.	(*c*)

Of these possible cases, if (*a*) be true, then by Rule iii. the I proposition, "Some dogs are black," is also true. Or if (*b*) be true, then the O proposition, "Some dogs are not black," follows in like manner. And if we find (*c*) to be true, then both the I and the O are true also. Hence we have the law—

(iv.) **Of two Sub-contraries, both may be true, but both cannot be false.**

The definitions given above of the various kinds of opposition have been summed up in the subjoined figure, with which the student should make himself perfectly familiar, so as to be able to reproduce it from memory. The laws i.—iv. of those oppositions are also summarized in the formulæ which follow the figure; and the student is recommended to prove for himself each of these formulæ by assigning to it its proper law.



If A be true; E is false, I true, O false.
If A be false; E is unknown, I unknown, O true.

If E be true; A is false, I false, O true.
If E be false; A is unknown, I true, O unknown.
If I be true; A is unknown, E false, O unknown.
If I be false; A is false, E true, O true.
If O be true; A is false, E unknown, I unknown.
If O be false; A is true, E false, I true.

Examination of these formulæ will enable us to express some of the laws of opposition in more convenient forms than those above. Thus :—

Of **contraries,** *the affirmation of one gives a right to deny the other, but the denial of one gives no right to affirm the other. Of* **contradictories,** *affirmation of the one is denial of the other, and denial of the one is affirmation of the other.*

And the results of the formulæ may, as a whole, be summed up in the following rule :—

The affirmation of a universal proposition, and the denial of a particular one, enable us to affirm or deny all the other three; but the denial of a universal proposition, and the affirmation of a particular one, leave us unable to affirm or deny two of the others.

"It is a rule of practical Logic that a contradictory should always in disputations be used in preference to a contrary opposition; for it serves equally well the purpose of contradicting an opponent, and the particular proposition which it asserts affords less ground for an attack than an universal. Thus, if my opponent asserts A (as, *e.g.*, All philosophers

are unimaginative), I may meet his assertion by the contradictory O (Some philosophers, as, *e.g.*, Plato, Goethe, etc., are not unimaginative), and from this position I cannot well be dislodged. But suppose I assert, in opposition to him, an E proposition (No philosophers are unimaginative), he will probably be able to adduce instances of some philosophers who, according to the ordinary meaning of the word 'imaginative,' would be called unimaginative, and so, by meeting my E with an I proposition, gain an apparent victory. As a fact, we should each have made assertions too wide; but he would have succeeded in dislodging me from my position, whereas (owing to my neglect of the laws of contradiction) I should not have succeeded in dislodging him from his." *

CONVERSION.

Definition.—A proposition is said to be *converted* when its terms are transposed, so that the subject becomes the predicate, and the predicate the subject. The result of this process is called *the converse* of the original proposition.

Thus:—

Proposition—Some Englishmen are sailors.
Converse—Some sailors are Englishmen.

In considering this process, we shall soon find ourselves beset by a difficulty. Take, for instance, the proposition—

All carrots are vegetables.

* *Elements of Deductive Logic.* By Professor Fowler. Fourth Edition, page 75.

Converted in the same way as the instance above, we get the converse—

All vegetables are carrots.

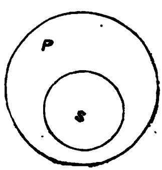
Now, between the two instances we have here there is this very considerable difference; that in the first case the converse was as true as the original proposition or *convertend*, while in the second case the convertend is true and the converse false. This difficulty can only be explained when we have examined the question of the *Distribution* of the terms in a proposition.

Distribution.—Let us examine the four forms of proposition in order.

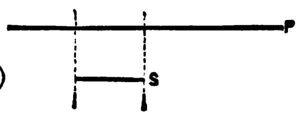
(A) All S is P.

Now, it is clear that we here, in this "universal" proposition, speak of *the whole* of the class which is denoted by S. But we do not say whether the class S forms *part* or *all* of the class P. Symbolically, we may represent this in any of the three ways, (α), (β), (γ):—

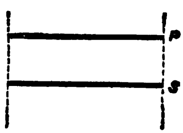
(α)



(β)



(γ)



In (α) and (β) all the Ss form part of the Ps; in (γ)

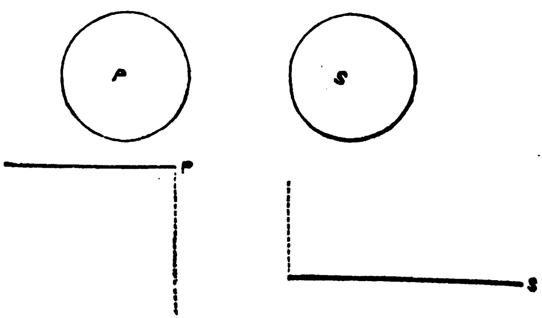
all the Ss form all the Ps. Thus, in the proposition, "All S is P," it is only said that all the Ss are included under the Ps, but whether there are any Ps which are not Ss is not stated. All the proposition says is that all the Ss form *some* part of the Ps; how large a part we do not know. Acting, then, on the principle above given, that a proposition must in logic always be understood to mean exactly what it says, ***and no more***, we find ourselves bound to understand the general meaning of an A to be—

All S is some P.

Now, when a proposition speaks of the whole of a term, that term is said to be *Distributed;* when the proposition speaks of part only of a term, that term is said to be *Undistributed.* From our example we can derive the rule that an A proposition distributes its subject, and leaves its predicate undistributed.

(E) No S is P.

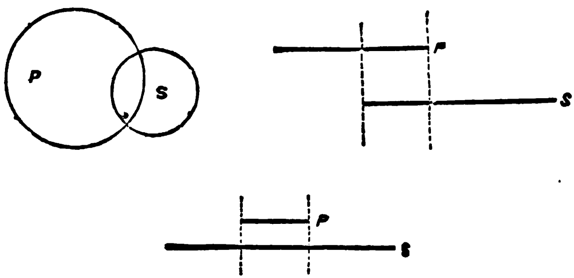
This proposition can be represented graphically thus :—



Here the whole of the Ss lie outside the whole of the Ps. Both terms are therefore spoken of in their entirety. Hence the rule, that an E distributes both its subject and its predicate.

(I) Some S is P.

Here it is evident that neither term is distributed. For the proposition only affirms that some portion of the Ss constitute some portion of the Ps, an affirmation which we may symbolize thus—



Some of the Ss may constitute either part of, or all of, the Ps. We must therefore take the general form of an I to be,—

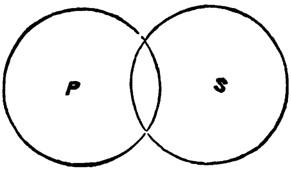
Some S is some P

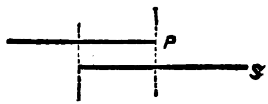
where both are undistributed.

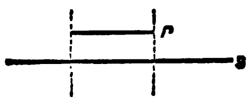
(O) Some S is not P.

Here a portion of the Ss is excluded from the whole

of the Ps. The proposition in reality says, "You may take *all* the Ps, but there will still be some Ss which







you will not have." Hence the rule in this case is, that an O proposition distributes its predicate but not its subject.

For convenience of reference, we may arrange these four rules thus:—Let D = distributed, and U = undistributed. Then we have

Proposition	Subject	Predicate
A	D	U
E	D	D
I	U	U
O	U	D

From this table we may see at a glance that whilst all negatives distribute the predicate, all affirmatives leave the predicate undistributed.

Now for legitimate, or "*illative*" conversion, there is

a law of the very highest importance, a law easily assented to when explicitly stated, but which it is only too common in the ordinary matters of life to neglect.

Law for Conversion.—*The converse, which must be of the same quality as the convertend, must not distribute any term which was not distributed in the convertend.*

Let us apply this law to the four propositions.

(A) All S is P.

Here S is distributed, P not. If, then, we convert into All P is S, the term P, undistributed in the convertend, will be distributed in the converse. We must, therefore, to get a legitimate converse, *limit* the distribution of P, and take for converse

Some P is S,

thus converting an A into an I. This kind of conversion, where the converse is of less quantity than the convertend, is known by the Latin name of Conversion *per accidens.* The student may call it if he pleases *Conversion by Limitation.*

We may now, therefore, solve the difficulty which met us on p. 35. The A proposition, "All carrots are vegetables," has for illative converse, *not* "All vegetables are carrots," but "Some vegetables are carrots."

(E) No S is P.

Here both terms are distributed. We may, therefore convert at once into

No P is S.

This kind of conversion, where the converse is of

like quantity with the convertend, is called *Simple Conversion.*

(I) Some S is P.

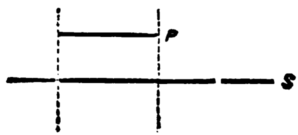
Here neither term is distributed. We can, therefore, again convert simply—

Some P is S.

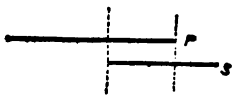
(O) Some S is not P.

This case will require a little more examination. We may represent the proposition graphically in either of the two ways (1) and (2) :—

(1)



(2)



In either of these cases some of the Ss lie outside the Ps, but there is this difference, that in (1) none of the Ps are outside the Ss, whilst in (2) some of the Ps are inside and some outside the Ss. In *both* cases there is some portion of the Ss which is outside all the Ps.

Now these considerations will show that it is impossible to convert an O proposition at all. For the existence of the case represented in (1) will show us that we cannot say

Some P is not S

as the legitimate converse, for in this case all P is S.

Thus, there is no illative converse to an O proposition.

We may, perhaps, make this clearer by considering a concrete case. Compare the two propositions.

(a) Some animals are not dogs.
(b) Some dogs are not black.

Both of these are O propositions, but in *(a)* the predicate "dogs" is a species of the genus "animal"; in *(b)* there is no such relation. We cannot, of course, convert *(a)* to "some dogs are not animals." And as we can only depend on the form and not at all on our knowledge of the subject-matter for our logical processes, we can never with safety convert an O proposition.

The full bearing and importance of the rule for the conversion of a universal affirmative proposition will be seen more and more clearly as the student progresses. It must be distinctly understood that when we have established the truth of an A proposition, we have not established the truth of its simple converse. I may prove to the satisfaction of everyone that "All S is P"; but if I then wish to prove in addition that "All P is S," I must start afresh from the beginning, and establish this latter proposition by another complete chain of reasoning. For instance, I may prove by geometry the proposition

All equilateral triangles are equiangular triangles;

but from this I know nothing as to whether or not

All equiangular triangles are equilateral triangles;

to prove which I must start entirely afresh. Another instance of this relation, and one which will be early met with in reading Geometry, is contained in Euclid

I. 5 and 6. In the first of these demonstrations, it is proved in effect that

All triangles with two equal sides are triangles with two equal base angles.

When, however, this is completely established, it is still necessary to commence again, and prove by a separate process that

All triangles with two equal base angles are triangles with two equal sides.

OBVERSION.

The process of *obversion* can now be investigated. It will be found to be very simple if we once clearly grasp what is exactly meant by the *opposite* of a proposition.

The opposite of a Term may be arrived at by prefixing or affixing a negative particle to it, or by any process of similar effect. For instance, the terms Mortal, Coloured, Dependent, Inside, have for opposites—Immortal, Colourless, Independent, Outside. Some pairs of opposites express notions so very common and familiar, that each member of the opposition is expressed by a separate word, as in Large, Small; Light, Dark; Light, Heavy; Long, Short; Straight, Crooked; Wise, Foolish, &c. We must here, however, recall what we said when treating of opposition, that there may be many alternatives between a term and its opposite; thus a thing may be neither light nor heavy, but somewhere between the two; a man who is no longer young need not at once be considered old, etc. And when each member of the opposition is expressed by a separate word, it will generally be

found that the intermediate stages have been neglected, and that, therefore, whilst the affirmation of one member gives us a right to deny the other, the denial of one gives no right to affirm the other. Such opposites may be called Contrary Terms. Very few pairs of simple terms are what is called contradictory opposites; that is to say, opposites in a sense permitting of the denial of one carrying with it the affirmation of the other. Alive, Dead; True, False; Right, Wrong (as applied to the result of a calculation) are such pairs of opposites:—the student may, as an exercise, try to find some more. He will find, however, that, as a rule, the contradictory opposite of a simple term is complex—as in such a case as Black and Not-black.

The opposite of a Proposition is formed by putting in the place of the Predicate the contradictory opposite of that Predicate.

Thus take the proposition,

The dog is alive.

The opposite of the term "alive" is the term "dead." Therefore the opposite of the given proposition is—

The dog is dead.

The student will by this time have recognized that the opposite of a proposition is only its contrary formed in another way (Cf. p. 31.) The opposite of the proposition "All dogs are animals," is "All dogs are not-animals." Here the proposition retains its original quality, and the sign of negation is attached to the predicate. If the negation had been attached to the subject instead of to the predicate of the original proposition, a change of quality would have occurred, and we should have had—

No dogs are animals,

which, as we have seen above, is an E proposition, and the contrary of the given A proposition.

Thus the Opposite and the Contrary of a proposition both deny everything that that proposition affirms; but the Opposite is of the same quality as the original proposition, the Contrary is of different quality. Both mean the same.*

For another instance, examine the two propositions—

All Japanese are not-Europeans; (A)
No Japanese are Europeans. (E)

These two propositions have both the same meaning.* The first is the Opposite, the second is the Contrary, of the proposition,

All Japanese are Europeans. (A)

We may now proceed to develope the rule given above (p. 34), that to affirm a proposition is to deny its contrary. Having found that the Contrary of a proposition is the same as its Opposite, we may alter the rule, and say, that to affirm a proposition is to deny its opposite. We then have

Definition.—When, instead of asserting any proposition, we deny its opposite, the proposition thus formed is called the *Obverse* of the original proposition.

---

* Any distinction of meaning that may be drawn between them belongs rather to Metaphysics than to Logic, and need not be considered here.

Thus, given an A proposition,

All bodies are coloured;

its obverse will be,

No bodies are colourless. (E)

The E proposition,

No fictions are true,

has for obverse,

All fictions are untrue. (A)

The I proposition,

Some men are rich,

may be obverted to,

Some men are not poor. (O)

The O proposition,

Some of these sums are not right,

has for obverse,

Some of these sums are wrong. (I)

CONTRAPOSITION.

Definition.—If of a proposition (*a*) the quality is reversed, (*b*) the negation of the predicate made the subject, and (*c*) the original subject made the predicate, the resulting proposition is called the contrapositive of the original proposition.

Thus, taking the A proposition,

All S is P,

its contrapositive will be,

No not-P is S. (E)

This contrapositive is perhaps more often used in its obverse form,

All not-P is not-S.

For example, having proved (Euc. I. 6) that every triangle with two equal base-angles has two equal sides opposite to them, we know at once that every triangle with unequal sides has unequal base-angles, or that no triangle with unequal sides has equal base-angles.

The contrapositive of the E proposition,

No S is P,

must be obtained by limitation (p. 40), and becomes

Some not P is S. (I)

The contrapositive of an I proposition cannot be obtained at all. For of the four representations on the next page, it is clear that the I proposition, "Some S is P," may be applied to all, even though in the two upper ones the universal, "All S is P," is also represented (Cf. p. 32, iii.). Now the contrapositive form, "Some not-P is not S," would be always true by the first three figures, but would become false by that marked (*b*) in every case where the line S is of indefinite length, as representing the sum-total of existence. Hence there is no evidence either way for or against the truth or falsity of the contrapositive of an I proposition, granting the truth of the I itself.

The O proposition,

Some S is not P,

has for contrapositive,

Some not-P is S. (I)

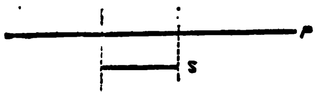
COMPATIBILITY.

Definition.—Two propositions are said to be **compatible** when, containing the same terms, the truth or falsity of the one does not in any way imply the truth or falsity of the other.

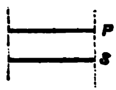
Take the two propositions—

(1) All S is P and Some P is not S. (2)

Of these, if (1) is *true*, (2) may be either true or false. For if (1) means that all the Ss form a part, and a part only of the Ps, then (2) is true, as thus—

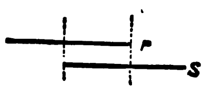


But if the meaning of (1) be that all the Ss form all the Ps, then (2) is false, thus—

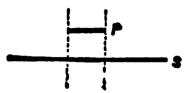


Hence we see that the *truth* of (1) does not imply either the truth or the falsity of (2).

Now let (1) be *false*. Then since it is false that all S is P, its contradictory, Some S is not P, must be true. (Law of Contradictories, p. 32). This we can represent in the two ways—



(*a*)



(*b*)

Of these, if (*a*) represent the real state of the case, it

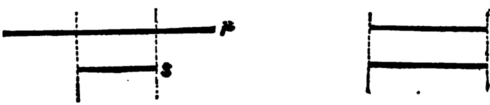
is *true* that Some P is not S; if (*b*) is the correct representation, then it is *false* that Some P is not S.

Hence we see that either the falsity or the truth of "All S is P" may be combined with either the falsity or the truth of "Some P is not S;" in other words, that these two propositions are perfectly compatible.

Again, take the two propositions—

(1) All S is P and (3) All P is S

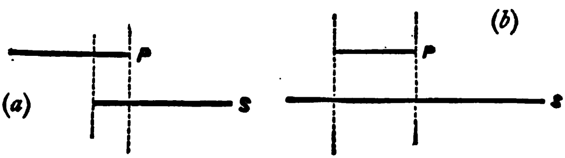
and first let (1) be *true*. It may be represented, as before, in the two ways—

(*a*) 

(*b*)

Here (*a*) makes it *false* that all P is S, while (*b*) makes it true. Thus the truth of (1) does not imply the truth or falsity of (3).

Next, let (1) be *false;* then, that "Some S is not P" is true (p. 32). This, as before, has the two representations—

(*a*) 

where from (*a*) it is *false* that All P is S, while from (*b*) it is *true*. Thus the *falsity* of (1) is consistent with either the truth or falsity of (3). Putting together these results into one phrase, we say that the proposi-

tion "All S is P" is compatible with either of the two propositions, "All P is S" and "Some P is not S."

Now it is clear that with the two terms S and P, we can only form eight propositions—A, E, I, O, with S for subject and P for predicate, and A, E, I, O, with P for subject and S for predicate. The student may examine each of these in detail, and will arrive at the result that of these eight propositions those examined above are the only ones which are, in the sense laid down in the definition on p. 48, compatible with the proposition "All S is P" *

We have now completed the investigation of the received logical changes of the proposition. For convenience of reference, we may here draw up a complete table of these changes as applied to the A proposition, and the student is recommended to make for himself similar tables as applied to the E, I, and O propositions.

* It may be well to give notice here that the term *compatible* will be met with by the student in the course of his logical studies as used in a different sense. Whenever the truth of one proposition does not imply the falsity of another, the two are sometimes said to be compatible. In this sense of the word subalterns are compatible. But I believe there is great convenience in restricting the meaning of the term as in the text.

And it should further be noticed that the compatibility of terms stands on a different footing from that of propositions. Any two terms, between which there is no logical connection, which are distinct without being contradictory, are called compatible terms. Such are "blue" and "heavy," "fair-haired" and "good-tempered," "cube" and "crystal," etc.

Proposition:—

All S is P. (A)

Contrary:—

No S is P. (E)

Contradictory:—

Some S is not P. (O)

Subaltern:—

Some S is P. (I)

Converse:—

Some P is S.

Obverse:—

No S is not-P.

Contrapositive:—

No not-P is S.

(or) All not-P is not-S.

Compatibles:—

All P is S. ....(Inverse)

(and) Some P is not S.

Reciprocal:—

All not-A is not-B.

CHAPTER VIII.

OF KINDS OF INFERENCE.

Definition.—Whenever, as a consequence of one or more given propositions (premisses), we assert or believe another proposition (conclusion), the combination of premisses and conclusion is called an **Inference.**

Of inferences we have more than one kind. We may state a proposition such as "All S is P," and

from it *alone* deduce certain other relations between P and S, such as that "Some P is S," etc. Such an inference is called an *Immediate Inference*. Or we may compare both P and S with some third thing, M, and from their relations to this third *infer* their relations to one another; as, for example—

M is the same size as P,
S is the same size as M,
∴ S is the same size as P.

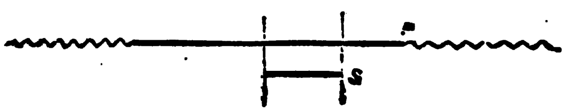
Such an inference is called a *Mediate Inference*, the two terms of the conclusion being brought into relation by aid of the intermediate third term.

We shall now have little difficulty in investigating the number and nature of the Immediate Inferences possible from a single proposition. Take, for example, the A proposition, and suppose it *granted as true* that

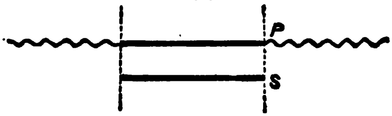
All S is P. . . . . (A)

This proposition may be represented graphically in either of the ways (1) and (2), where the waved line, supposed to be of indefinite extent, may be taken to represent the negative term "Not-P."

(1)



(2)



Now, from either of these figures we can at once read off the four propositions—Some P is S, Some S is P, No S is not-P, and No not-P is S. These are respectively the Converse, the Subaltern, the Obverse, and the Contrapositive of the original A proposition.

We shall also be able to find the following propositions justified either by one or by both together of our figures:—

> "The proposition 'No S is P' is false."
>
> "The proposition 'Some S is not P' is false."
>
> "The truth or falsity of the proposition 'All P is S' is unknown."
>
> "The truth or falsity of the proposition 'Some P is not S' is unknown."

Of these, the first two follow at once, from either figure. The third follows from the fact that the proposition in question, "All P is S," is false by (1) and true by (2). The ground for the fourth is similar, "Some P is not S," being true by (1) and false by (2).

This investigation will lead us to recognize the fact that we can make with a single proposition eight immediate inferences, as follows:—

[TABLE

1.	(A) (I) ∴	All S is P, Some P is S.	an Immediate Inference called . . .	Conversion.
2.	(A) (I) ∴	All S is P, Some S is P.	" " . . .	Subaltern Opposition.
3.	(A) (E) ∴	All S is P, No S is not-P.	" " . . .	Obversion.
4.	(A) (E) ∴ (A)	All S is P, No not-P is S, *or* All not-P is not-S.	" " . . .	Contraposition.
5.	(A) (A) ∴	All S is P, The proposition 'No S is P' is false.	an Immediate Inference as to	Contraries.
6.	(A) (A) ∴	All S is P, The Proposition 'Some S is not P' is false.	" "	Contradictories.
7.	(A) (A) ∴	All S is P, The truth or falsity of the proposition 'All P is S' is unknown.	" "	Compatibles.
8.	(A) (A) ∴	All S is P, The truth or falsity of the proposition 'Some P is not S' is unknown.	" "	

It should be carefully noted that we have here four immediate inferences of one kind, and four of quite another. In inferences 1-4 the terms of the conclusion are in substance the same as the terms of the premiss; in inferences 5-8 they are quite different. In the last four cases the conclusion is, in fact, of a complex character, it is *a proposition about a proposition.* Thus, to compare, for example, inferences 2 and 6. In 2 the subject and predicate of both premiss and conclusion are respectively S and P. In 6 the subject and predicate of the premiss are S and P respectively, but the subject of the conclusion is "The proposition 'some S is not P,'" and the predicate of the conclusion is "False."

We thus see that from an A proposition we can at once directly infer its Converse, Subaltern, Obverse, and Contrapositive, each of these having in substance the same terms as the original proposition. These are Immediate Inferences of the First Kind. We cannot directly infer the Contrary, the Contradictory, or the Compatibles, though we can make inferences as to their truth or falsity, such inferences being Immediate Inferences of the Second Kind. The conclusion of such inferences will always, as shown in the table above, be an A proposition.

CHAPTER IX.

OF MEDIATE INFERENCE OR SYLLOGISM.

It is the essential property of all mediate or syllogistic inferences that there should be two things

whose respective relations to *the same* third thing are given, whence we can infer their relation to one another. It is obvious that if two things resemble a third in any particular, they must also resemble each other in that particular. Thus, if A and B are both of the same size as X, clearly A and B are themselves of the same size as one another. From this example, and from those given in Chap. I., it follows that there can only be three terms in a syllogism, and that there must be three; while an exactly similar rule holds as to the number of propositions of which a syllogism may consist. For it would be idle to endeavour to prove that A and B were of the same size by saying that A was the same size as X, and B of the same size as Y. In so simple a case such a mistake would be obvious, and the argument containing it would be at once rejected, but the logical consequences are exactly the same whenever the middle term, or term with which each of the other terms is compared, is used in an ambiguous sense. For, logically, an ambiguous term is not one, but two. Thus, we should never accept such an argument as

A file is a tool,
A row of soldiers is a file,
∴ A row of soldiers is a tool.

Such an argument clearly contains four terms, though the two different terms *file* happen to have the same sound. Symbolically the argument stands thus:—

M is P,
S is N,
∴ S is P.

where the conclusion finds no warrant in the premisses, and where the premisses in fact lead to nothing. Thus, a syllogism must contain three, and only three terms.

An inference, which is in any way defective, is called a **Fallacy**. The argument above examined is a case of what is known as the fallacy of "**Ambiguous Middle.**"

It is equally necessary that a syllogism consist of three and only three propositions. Two of these must be given us to start with, and are occupied in stating the relation which each term of the conclusion bears to the middle term; the final proposition then stating explicitly what is the relation between those two terms themselves. In this way we arrive at the general analysis of every syllogism :—

Every syllogism must consist of three and not more than three propositions, containing three and not more than three terms.

Thus, repeating again the example of Chapter I.—

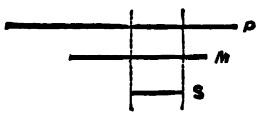
All birds are bipeds	Premisses
All thrushes are birds	
∴ All thrushes are bipeds. . . .	Conclusion

we have given us the two propositions called Premisses. These have the common term "birds," and from these two we infer a third proposition, called the Conclusion, which brings into relation the remaining term "thrushes" of the one premiss to the remaining term "bipeds" of the other premiss. We know already that the term "bipeds," which is the predicate of the conclusion, is called the *major term*, and the term "thrushes," the

subject of the conclusion, is the *minor term.* The premisses are named after the terms which they respectively contain; that premiss which contains the major term being called the Major Premiss, and that which contains the minor term being called the Minor Premiss. The term which is common to the two premisses, but which does not appear in the conclusion, is the *Middle Term;* and the Major and Minor Terms being brought by the Major and Minor Premisses, respectively, into relation with the same Middle Term. are thus brought into relation with one another.

So far our analysis was carried in Chapter I. We found there also that the relation between the terms of the premisses was one of containing and contained. In the syllogism

All M is P,
All S is M,
∴ All S is P.



we assert that the class M is contained in the class P, and the class S is contained in the class M, and that thus we know that S must be contained in the class P. It only remains to investigate the rules under which such inferences can legitimately be drawn.

We must, however, first notice that in each of our two premisses, the middle term can stand either as subject or predicate, and, when we have decided its position, the choice will still remain as to the quantity and quality of the premisses themselves. If we let M be the middle term, P the major term, and S the minor term, we shall have all possible major premisses thus—

	A	E	I	O
(α)	All M is P	No M is P	Some M is P	Some M is not P
(β)	All P is M	No P is M	Some P is M	Some P is not M

and all possible minor premisses thus :—

(γ)	All S is M	No S is M	Some S is M	Some S is not M
(δ)	All M is S	No M is S	Some M is S	Some M is not S

and as far as it is a matter of arrangement only, and neglecting for the present any question as to the validity of the resulting syllogism, we can unite any one of the minors to any one of the majors to give us the two premisses which every syllogism must contain.

Now, according to the position of the middle term in the premisses, the syllogism is said to vary in **Figure.** The middle term may be made the subject of the major premiss [by selecting the major premiss out of line (α)] and the predicate of the minor premiss [by selecting the minor premiss out of line (γ)] and the resulting syllogism is said to be of the **First Figure.** Or the middle term may be predicate in both premisses [by selecting the major from line (β) and the minor from line (γ)] and the resulting syllogism is said to be of the **Second Figure.** Or, again, the middle term may be made the subject in both the premisses [by selecting the major from line (α) and the minor from line (δ)], and the syllogism thus obtained is said to be of the **Third Figure.** Lastly, we may reverse the order of the first figure, and put the middle term as predicate of the major premiss and subject of the minor [selecting the major from line (β), and the minor from line (δ)], thus constructing a syllogism of the **Fourth Figure.** It is evident that

no other arrangements are possible, and that the four above enumerated constitute the whole of the logical figures.

Another distinction is caused by the quantity and quality of the premisses we use in the formation of our syllogism. For when we have selected the lines from which our premises are to come, we may still find each of those premisses taking any one of the forms A, E, I, O. For instance, supposing we have decided to throw our reasoning into the first figure, the premisses will be selected from lines (α) and (γ). Now, it is clear that with each of the four major premisses can be combined any one of the four minor premisses, and that thus the whole number of combinations will be sixteen, as follows:—

AA, AE, AI, AO, EA, EE, EI, EO, IA, IE, II, IO, OA, OE, OI, OO.

According to the quantity and quality of the propositions of which it is composed, the syllogism is said to vary in **Mood**. From the above number of possible premisses we see that there are sixteen *possible* moods to every figure.

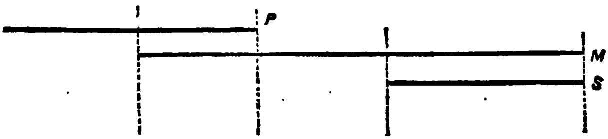
But though all the sixteen moods are *possible* in every figure of the syllogism, it does not follow that every mood is in all cases *legitimate* in each figure. In fact, it will soon be found that the number of legitimate moods is limited to comparatively few by the operation of those rules which we have now to investigate, and which are the ultimate laws of all syllogistic reasoning.

We will premise that in accordance with the plan and limits of the present volume, it is only intended to examine syllogisms of the First Figure.

I. Let us examine an argument in the following form :—

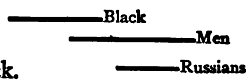
Some M is P,
All S is M,
∴ Some S is P.

We shall find that the conclusion here is not justified by the premises. For the argument may be graphically represented thus :—



and here it is evident that there is no necessary relation between S and P, for the minor term may belong to one part of the middle term, while quite another part of the middle term belongs to the major. The falsity of this argument will become even more apparent if we take an example the meaning of whose terms we know. For instance—

Some men are black,
All Russians are men,
∴ Some Russians are black.



Here we clearly see that the Russians belong to a portion of men which is not included in the class *black*, and hence the inference is erroneous. The fault was that we were able to divide the middle term into two parts, and so, making use of a different part in each premiss, we failed altogether to bring the major and minor terms into any relation with one

another. The major and minor terms can be brought into relation with one another by means of the relation of each of them to *the same* middle term; here they are not really brought into relation with the same middle term at all, since one belongs to one part of it, and the other to another.

This error could not arise if the middle term was distributed once in the premisses. In the premisses as they stand the middle term is not distributed at all, for in the major premiss it is a *particular subject*, and thus undistributed by nature; in the minor premiss it is the *predicate of an affirmative*, and, therefore, undistributed by position (Cf. p. 39). And unless the middle term is at least once distributed, we shall always be unable to draw any certain conclusion from our premisses. Hence we have

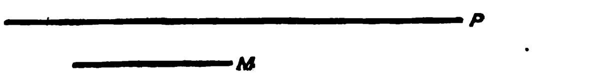
Rule I.—**The middle term must be distributed at least once.**

The fallacy above examined is called the fallacy of **Undistributed Middle.**

II.—Suppose we have presented to us an argument in the form—

All M is P,
No S is M,
∴ No S is P.

The premisses may be represented graphically thus:—



And here it is clear that the conclusion, No S is P, is not represented, and that, so far as our premisses inform us, either all or some of S may be P.

It remains to determine where the fault lies. In the major premiss, all M is P, we declare that all the Ms form some part of the Ps, but we do not say *what* part—*i.e.*, whether part or the whole. In other words, the major term is undistributed in the premisses. Now, in the conclusion, No S is P, it is affirmed that the whole of the Ss are excluded from the whole of the Ps; that is to say, the major term is distributed in the conclusion. This, then, will give us the origin of our error—that in our conclusion we use the whole of a term, of which in our premisses only a part was given to us. We may take, as a concrete example—

All horses are animals,
No dogs are horses,
∴ No dogs are animals.

The absurdity of such an argument is evident at a glance, and hence we have our second syllogistic rule:—

Rule II.—**No term must be distributed in the conclusion which was not distributed in the premisses.**

The breach of this rule gives rise to a fallacy known as **Illicit Process.** Illicit processes are of two kinds, according as the major or minor term is wrongfully distributed in the conclusion. In the former case, the fallacy is called Illicit Process of the Major; in the latter, Illicit Process of the Minor Term.

Illicit process of the minor does not occur in prac-

tice so often as illicit process of the major. An example of it would be—

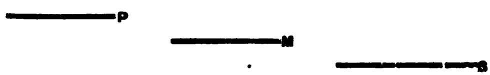
All birds are winged,
Some bipeds are birds,
.·. All bipeds are winged.

It will be easily seen that this rule has already been investigated in the case of immediate inference, and is the same law in another form as that which regulates conversion by limitation. (Cf. p. 40.)

III.—Suppose we have given us as premisses—

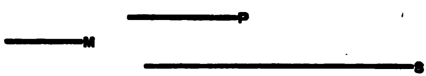
No M is P,
No S is M.

We can represent these graphically, thus:—



or thus:—

or thus:—



From these figures we may gather that any variety of relation between S and P is consistent with the premisses. This might, in fact, be seen at a glance from the premisses themselves, for all they do is to deny any relation between the major and minor and the

middle term. Hence they leave the relations between the major and the minor wholly undetermined.

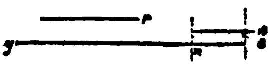
In this way we arrive at

Rule III.—**Two negative premisses prove nothing.**

IV.—Having examined the case where *both* premisses are negative, it remains to be seen what law will hold in the case where one only of the premisses is negative. Take, for instance—

No M is P,
Some S is M.

This can be represented thus:—



Now, the only portion of S that we know anything about is that portion which is contained under M. Therefore the premisses will be equally well represented whether the line S stops at x or is produced to y. Thus, since all the M lies outside P, it follows that all the S of which we have any knowledge also lies outside P; that is to say, that our conclusion from the above premisses must be negative—

Some S is not P.

A still stronger case would have been made if our minor premiss had been a Universal, instead of a Particular Affirmative:—

No M is P,
All S is M;

for then we should have known that in our figure the line S could not be prolonged beyond the point x, and our conclusion would have been—

No S is P.

Hence, whether the negative premiss be universal or particular, we shall always have to observe the following rule :—

Rule IV.—**If one premiss be negative, the conclusion must be negative.**

V.—A further examination of Case IV., above, will give us another rule, thus :—

Rule V.—**If the conclusion be negative, one premiss must be negative.**

For the only way to prove that S has no kind of connection with P is to show that either the minor or the major is wholly contained in some third term, M, which itself is known to have no connection with the major or the minor respectively.

VI.—Let us now examine the case where both premisses are particular. We must have either two O propositions, two I propositions, or one O and one I. We cannot by Rule III. have two O propositions for premisses, since two negative premisses prove nothing; nor can we have two I propositions for premisses, such as—

Some M is P,
Some S is M;

for such premisses would leave the middle term undistributed (Rule I.). Can we, then, have one of

each—an O and an I, or an I and an O? Trying first thus—

Some M is not P,
Some S is M,

we find the middle term still undistributed (Rule I.). We have, therefore, only the case left in which we have for premisses an I and an O, as thus:—

Some M is P,
Some S is not M.

Here we have complied with Rule I., since the middle term is distributed in the minor premiss. But we have now a negative premiss (the minor), and therefore (Rule IV.) the conclusion, if any, must be negative. A negative proposition distributes its predicate; and the major term is not distributed in the premisses. Therefore, the only form of conclusion left to us would involve illicit process of the major term (Rule II.). Hence we have—

Rule VI.—**Two particular premisses prove nothing.**

VII.—It remains to consider the case where one premiss only is particular. Let the given premisses be—

All M is P,
Some S is M.

Here neither S nor P is distributed. The only legitimate conclusion must therefore be—

Some S is P,

and this conclusion is particular.

In the above example the I proposition was com-

bined with an A. Let us now combine it with an E:—

No M is P,
Some S is M.

Here the conclusion must be negative (Rule IV.), and must not distribute the term S (Rule II.). Hence its only form can be—

Some S is not P,

and this conclusion is again particular.

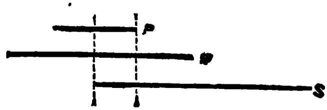
Next, let the one particular proposition in the premisses be an O. With this we cannot combine an E, because then both premisses would be negative (Rule III.), nor with an I, which would give us two particular premisses (Rule VI.). The other premiss must therefore be A:—

All M is P,
Some S is not M.

Here the conclusion must be negative (Rule IV.). But a negative conclusion distributes its predicate, and in the given premisses P is not distributed. Hence, in this case we can have no conclusion.*

* The combination of an A and an O will, however, have a legitimate conclusion in the second figure, where, it will be remembered, the middle term is predicate in both premisses, thus:—

All P is M,
Some S is not M,
∴ Some S is not P.



Here again, however, the rule still holds that the conclusion is particular.

Thus, in every case where we are able to obtain a conclusion by the use of a particular premiss, we have found that conclusion to be particular, hence, we have—

Rule VII.—**If one premiss be particular, the conclusion must be particular.**

The above seven rules are applicable to all syllogistic reasoning, into whatever form or "figure" it may be thrown. It remains to see whether any special conditions are imposed upon us by reasoning in the first figure—that is to say, by making our middle term the subject of the major premiss and the predicate of the minor. Take, for instance, the premisses,

All M is P;
No S is M.

Here the conclusion, if any, must be negative; but a negative conclusion distributes its predicate, and P is not distributed in the premisses. Therefore from these premisses no legitimate conclusion can be drawn. The same result would follow if we had an O instead of an E for minor premiss. Hence we have, specially for the first figure (for in the second and fourth it does not hold)—

Rule (a).—*The minor premiss must be affirmative.*

Again, take for major premiss either
(i) Some M is P or (ii) Some M is not P.

With (i) we can only combine either an A or an E (Rule VI.). We must thus either have

$$\left.\begin{array}{l}\text{Some M is P,}\\ \text{All S is M,}\end{array}\right\} \quad \text{or} \quad \left\{\begin{array}{l}\text{Some M is P,}\\ \text{No S is M.}\end{array}\right.$$

The first of these cases leaves the middle term undistributed, and may therefore be dismissed (Rule I.). The conclusion, if any, of the second must be negative (Rule IV.); and a negative, which distributes its predicate, would here involve an illicit process. Thus in no case can the major premiss in the first figure be an I proposition.

Can we then as in (ii) have an O for major premiss? By Rules III. and VI. the only possible minor premiss would be an A, thus—

Some M is not P,
All S is M;

and as this leaves the middle term undistributed, we find that the major premiss of a syllogism in the first figure cannot be an O proposition. Combining these two results, we have

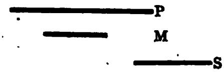
Rule (β).—*The major premiss must be universal.*

The general rules of the syllogism given above (I. to VII.) are those by which every syllogism must be tested. Every one of the sixteen possible moods (p. 60) may by their aid be examined for each figure, and accepted or rejected according as the inference is found to be valid or not. We will examine them in detail for the First Figure—

(i) A All M is P,
A All S is M,
A ∴ All S is P

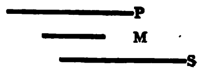
Valid.

(ii) A All M is P,
E No S is M,
E ∴ No S is P.



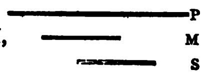
Illicit Process of Major.

(iii) A All M is P,
I Some S is M,
I ∴ Some S is P.



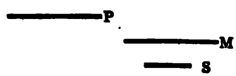
Valid.

(iv) A All M is P,
O Some S is not M,
O ∴ Some S is not P.



Illicit Process of Major.

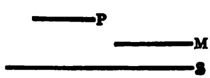
(v) E No M is P,
A All S is M,
E ∴ No S is P.



Valid.

(vi) E No M is P,
E No S is M,
No conclusion. (Rule III.)

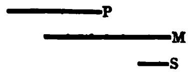
(vii) E No M is P,
I Some S is M,
O ∴ Some S is not P.



Valid.

(viii) E No M is P,
O Some S is not M,
No conclusion. (Rule III.)

(ix) I Some M is P,
A All S is M,
I ∴ Some S is P.



Undistributed Middle.

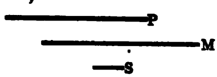
(x) I Some M is P,
E No S is M,
O ∴ Some S is not P.

Illicit Process of Major.

(xi) I Some M is P,
I Some S is M,
No conclusion. (Rule VI.)

(xii) I Some M is P,
O Some S is not M,
No conclusion. (Rule VI.)

(xiii) O Some M is not P,
A All S is M,
O ∴ Some S is not P.



Undistributed Middle.

(xiv) O Some M is not P,
E No S is M,
No conclusion. (Rule III.)

(xv)	O	Some M is not P,
	I	Some S is M,
		No conclusion. (Rule VI.)
(xvi)	O	Some M is not P,
	O	Some S is not M,
		No conclusion. (Rules III. and VI.)

We thus find that of the sixteen possible moods, we have been able to preserve as legitimate only the four—numbers i, iii, v, vii—AA, EA, AI, EI. These four comply with the special rules (α) and (β) of the First Figure, for in each of them the minor premiss is affirmative (Rule α) and in each the major premiss is universal (Rule β). The respective quantity and quality of the conclusions following legitimately from these premisses are found to be A, E, I, O, thus showing that in the first figure a proposition of any kind as regards quantity and quality can be proved. Adding the designating letter of the conclusion in each case to those of the premisses, we have as complete descriptions of the Moods of the First Figure the following :—

AAA, EAE, AII, EIO.

For more easily remembering these, there has been invented a mnemonic line of hexameter verse, where the vowels have been combined with consonants in such a way as to form words, which, while readily remembered, have also various other important uses in Logic. The line is—

b**A**rb**A**r**A**, c**E**l**A**r**E**nt, d**A**r**II**, f**E**r**IO**que prioris.

This line is of the utmost importance, and should be

carefully learnt by heart. No argument of the first figure can be valid unless it can be reduced to one or other of the four moods contained in it thus given. All other arguments in the first figure, that is, all other arguments whose middle term is subject of the major premiss and predicate of the minor, are fallacies.

To this Rule there are, however, two exceptions. It is possible to construct valid arguments of the first figure in each of the moods AAI and EAO, thus—

A	All M is P,	E	No M is P,
A	All S is M,	A	All S is M,
I ∴	Some S is P.	O ∴	Some S is not P.

Here the conclusion certainly follows from the premisses, but in each case the premisses are capable of proving, not only a particular, but a universal proposition. Premisses AA will prove an A conclusion, and premisses AE an E conclusion. Hence the syllogism in each of the above cases is said to have a *weakened conclusion;* and such a syllogism, though theoretically valid, is of hardly any practical importance.

Before leaving the subject of the syllogism, it will be well to point out that each figure is peculiarly liable to some one kind of fallacy. The fault which in practice most frequently finds its way into reasonings of the first figure is that of Illicit Process of the Major Term. We have already incidentally mentioned (p. 69) how this may happen in the course of our investigation of Rule (α), and it is by a violation of this rule that most of the fallacies in the first figure arise. For

instance, it might be reasoned with some show of plausibility, that, as all the Conservatives voted with the Government, and as Mr Gladstone is not a Conservative, he did not vote with the Government. An argument of this kind acquires a greater appearance of validity from our own knowledge of certain facts of political life, such as that, if all Conservatives voted with a certain Government, that Government would be probably itself Conservative, and, as such, would be one to which Mr Gladstone would find himself in opposition on a majority of points. And here it may be remarked that in practically estimating the logical value of any argument submitted to our examination, there is no greater danger, none that we need to guard ourselves against with more scrupulous care, than that of filling up an imperfect argument from our own knowledge, or of strengthening a weak one with the force of our own feelings. It is hard, for example, not to be far too easily convinced of the wrongdoing of a man whom we personally dislike.

The argument given above may be detected as fallacious in many ways. We may set ourselves to examine the case independently, and we may remark that it is not stated in the argument that the Government in question is Conservative at all. Allusion may be made to some case in which a moderate Liberal Government was supported by the Conservatives in opposition to extreme Radicals. The probabilities would then be on the side of Mr Gladstone's voting not against, but with, the Government. Such an examination of the argument itself is enough to show that it is not conclusive; but it will be finally over-

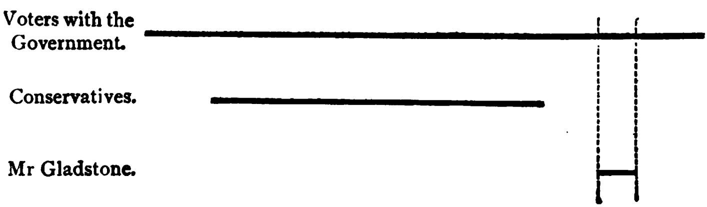
thrown when we investigate it according to the rules and methods hitherto laid down. Expressed as a syllogism, it stands thus—

A   All Conservatives are voters with the Government.
E   Mr Gladstone is not a Conservative.
E ∴ Mr Gladstone is not a voter with the Government.

In symbols thus—

All M is P,
No S is M,
∴ No S is P.

To this argument, either in its symbolical or its concrete form, we can apply the test we have all along used :—



From this diagram, it is clear that, provided the line for "Mr Gladstone" is kept outside that for "Conservatives, we are at liberty, so far as the given premisses are concerned, to place it either within or without the line, "Voters with the Government." Finding thence that the argument is fallacious, we can look back for the cause of the fallacy, and we shall find

that the major term, "Voters with the Government," being in the conclusion the predicate of a negative proposition, is there distributed; whilst in the major premiss it is the predicate of an affirmative proposition, and therefore undistributed. Hence the argument is guilty of an Illicit Process of the Major Term.

It is impossible to impress too strongly upon students who desire to pursue the study of Logic, the importance of careful practice in thus reducing arguments to a strict syllogistic form for examination, either by applying to them the test of a method of notation, or by the application to them of the syllogistic rules, one by one.

CHAPTER X.

OF OTHER METHODS OF DEMONSTRATION.

We have first to explain the sense of the word *Demonstration.* Properly it means a pointing out of the connection between the premisses and the conclusion of any argument (Latin, *Demonstrare,* to show). The word, however, has been generally received in a somewhat wider sense, and is understood to apply to the whole process of the reasoning, whatever may be its nature or its length, by virtue of which any proposition is held to be proven.

Now we have hitherto only dealt with propositions of simple construction, and with syllogisms composed

of such simple propositions. Every simple affirmation of a fact is a simple proposition, and such are called in logical language, **Categorical Propositions.** But the fact that we wish to assert, may be that the truth of one proposition follows from the truth of another. This fact, though itself as simple as any other fact, nevertheless finds its most convenient expression in a proposition of complex form. For instance, instead of saying

The truth of the proposition "C is D," follows from the truth of the proposition "A is B,"

I may much more shortly and conveniently express my meaning thus—

If A is B, C is D.

In this latter form two propositions are united into one, and the relation between them, which finds such a very condensed expression, is that, if the first is true, the second must be true also. Such a proposition is called a **Hypothetical Proposition.**

Of two propositions thus united, the first, from which the truth of the second follows, is called the **Antecedent**; and the second, the one whose truth follows from that of the antecedent, is called the **Consequent.** In our example, "A is B" is the antecedent; and "C is D" is the consequent. Now the exact nature of the relation between antecedent and consequent must be most carefully noted, for any want of clearness on this point will introduce serious fallacies into our reasoning. The proposition, then, asserts that if it be true that A is B, it will also be true that C is D. But from this we are not at liberty to conclude

that if it be false that A is B, it is also false that C is D. To take another instance—

If the prisoner is guilty, this witness is a liar,

does not imply the truthfulness of the witness if the prisoner is not guilty, for the witness may have been lying though the prisoner was innocent. As in the first instance, there may be many other conditions under which C is D besides the one stated, that A is B. Hence, we have the rule for hypothetical propositions, that to affirm the antecedent is to affirm the consequent, but to deny the antecedent is not to deny the consequent.

Again, in the hypothetical proposition, "If A is B, C is D," it is clear that if C is not D, A cannot be B. For the proposition says in effect that *every* case in which A is B, is also a case in which C is D. Or, all the cases in which A is B lie within the cases in which C is D. Hence a case which is not included amongst those cases in which C is D, cannot be amongst the cases in which A is B. This argument can be illustrated graphically in the same way as an ordinary syllogism, thus—

Cases in which C is D (i)
Cases in which A is B (ii)
Cases in which C is not D (iii)

Here it is evident that (iii) must be outside of (ii); or, referring again to our concrete example, we may see at once that if the witness be not lying, the prisoner must be innocent. Hence, to deny the consequent proves the falsity of the antecedent.

But the witness may be lying though the prisoner is not guilty. Thus to affirm the consequent proves nothing as to the truth or falsity of the antecedent.

In this way we obtain the complete canon for hypothetical reasoning:—**To affirm the antecedent, or to deny the consequent, proves respectively the truth of the consequent, or the falsity of the antecedent. To deny the antecedent, or to affirm the consequent, proves nothing.**

The rule thus obtained will enable us to form complete syllogisms with hypothetical propositions. Thus we may have an argument in a fully expressed form, thus:—

If A is B, C is D;
Now A is B,
∴ C is D.

Here the hypothetical proposition is the major premiss, and the minor premiss is the simple categorical statement that the condition has been fulfilled. Such a syllogism is called a **Constructive Hypothetical Syllogism.** It may easily be reduced to the categorical form, as follows:—

[All cases in which A is B] are [cases in which C is D].
[This case] is [a case in which A is B].
∴ [This case] is [a case in which C is D].

For greater distinctness the separate terms of the syllogism have here been enclosed in brackets, and it will be obvious that the form of it is

All M is P,
All S is M,
∴ All S is P.

which is AAA in the first figure, a legitimate syllogism.

So we can, in like manner, form hypothetical syllogisms in the cases where the denial of a consequent is used to disprove an antecedent. For instance:—

If A is B, C is D.
Now C is not D,
∴ A is not B.

This syllogism can also be reduced to a categorical form, though it does not fall into the first figure. In its hypothetical form it is called a **Destructive Hypothetical Syllogism.**

But Categorical Propositions and Hypothetical Propositions do not exhaust all the varieties of which the forms of assertion are capable. There are many assertions which state that some one of various alternatives is true. "That man is either a knave or a fool," is a proposition in which it is not asserted that the man is knave, nor that he is a fool, but only that if he is not one he must be the other. Such a proposition is called a **Disjunctive Proposition,** and since any number of alternatives from two upwards may be combined in one disjunctive proposition, we have as a general form for propositions of this kind—

A is either B or C or D or...........

Confining our attention for the present to disjunctives with only two alternatives, we shall find that of these alternatives the denial of either will always give the right to affirm the other. Thus, the example we have given above affirms that there are only two possible explanations of some phenomena under considera-

F

tion; if the man is not a fool he is a knave, and if he is not a knave he is a fool. We can thus have a **Disjunctive Syllogism** in the form—

A is either B or C,
But A is not B.
∴ A is C.

Here, it may be observed, that by denying one alternative we assert the other, and this kind of disjunctive syllogism is known by the name of *Modus tollendo ponens*, a Latin phrase meaning "the mood which by denying affirms." In such an argument the major premiss is a disjunctive proposition, and the minor a categorical negative. It must be noticed carefully that a disjunctive syllogism differs from a categorical syllogism in this very important particular:—that whereas in a categorical syllogism a negative premiss must be followed by a negative conclusion, in disjunctive syllogisms the opposite rule holds, and an affirmative conclusion follows from a negative premiss.

The *modus tollendo ponens* is always and unconditionally valid. The major premiss gives us all possible alternatives; and if we deny all but one, that remaining one must be true. But there is yet another possibility unaccounted for:—*more than one of the alternatives may be true at the same time.* Thus, when we say—

A is either B or C,

we have, as the proposition stands, no guarantee that A is not both B and C. "That man is either a knave or a fool," does not exclude the possibility of his being both.

Hence we can construct another kind of disjunctive argument which will not, however, have universal validity, thus—

A is either B or C,
But A is B.
∴ A is not C.

This form of the disjunctive syllogism is called the *modus ponendo tollens*, "the mood which by affirming denies." And this argument can only be valid where B and C are contradictory terms, mutually exclusive, such as *Equal* and *Unequal*, etc.; or any terms whose co-existence we know, as a matter of fact, cannot occur, such as *Black* and *White*, etc. But, in this latter case, we go beyond the range of pure logic, and are considering rather the matter than the form of our reasoning.

In either of the forms of the disjunctive syllogism we may have a major premiss which presents to us more than two alternatives. We shall then have to find either a minor premiss which, at one and the same time, deals with all of these alternatives but one; or else a succession of minor premisses dealing with them one after another till all but one are disposed of. For instance, we may either reason thus—

A is either B or C or D,
But A is neither B nor D,
∴ A is C.

or thus—

A is either B or C or D,
But A is not B,
∴ A is either C or D;
But A is not D,
∴ A is C.

Now it will often happen that such a major premiss is given us, containing a certain number of alternatives which we know independently to include all possible cases, while we also know each case to be inconsistent with any other. Such a major premiss, for instance, would be found in the proposition,

> Magnitude X is either greater than, equal to, or less than, magnitude Y.

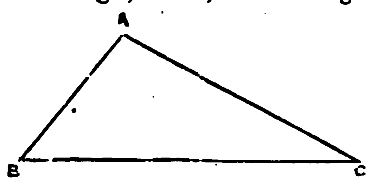
Here we know, independently of any special investigation, that we have all possible alternatives, and that no two of them are compatible. Hence, if to the above as a major premiss we can add the minor—

> Magnitude X is neither greater than, nor less than, magnitude Y,

we shall at once be able to conclude that—

> Magnitude X is equal to magnitude Y.

This kind of proof is frequently met with in Geometry. Thus, in Euclid I. 19 (Wilson's Geometry, I. 11), we have given us a triangle, A B C, with the angle at B greater than the angle at C, and it is asserted that the side A C will consequently be greater than the side A B.



The real framework of the process of reasoning gone through for the purpose of proving this assertion may be put as follows:—

Line AC is either less than, equal to, or greater than line AB ;
But line AC is not less than line AB [for certain reasons],
∴ Line AC is either equal to, or less than line AB ;
But line AC is not equal to line AB [for certain reasons],
∴ Line AC is greater than line AB.

Here our major premiss contains three alternatives which we know on grounds quite independent of geometry, to be incompatible with one another, and to be the only ones possible. Two of them being denied the third must be true. Such a method of proof, where in a disjunctive syllogism all possible cases are examined, and all rejected but one, which is thus left established, may be called **Proof by Exhaustion.**

Besides the regular syllogistic arguments, there are others which bear a very close outward resemblance to a syllogism, but which, when carefully examined, will be found not to be syllogisms at all, though perfectly valid as arguments. For example, we may have some such piece of reasoning as—

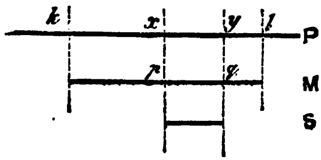
John is taller than William,
William is taller than Charles,
∴ John is taller than Charles.

A little examination will show that there is not in this argument any middle term. A middle term would appear if we could alter the argument into the following form :—

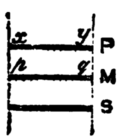
John is taller than William,
Taller than William is taller than Charles,
.·. John is taller than Charles.

But here our second proposition, the new minor premiss, is precisely the conclusion we had to obtain in our original argument, only in another form. And such an alteration is, of course, inadmissible, since it would be to assume in the premisses the very thing we have to prove. Let us then endeavour to find out in what respect this kind of argument differs from the ordinary syllogism.

All M is P,
All S is M,
.·. All S is P.



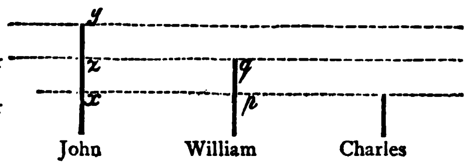
Here we have a syllogism with its representation. We see in this case that All M is P, and there is other P besides which is not M, viz., all the P which lies beyond the limits *k l*. But with these latter portions of P the syllogism has no concern; they may not even exist at all. Similarly the syllogism is not concerned with those portions of M which lie beyond the limits *p q*. And the conclusion takes no cognizance either of any M beyond *p q*, or of any P beyond *x y*. Thus the same syllogism would also be represented by the form



where in each case the excess of the containing term over the contained does not appear. In the syllogism therefore *we reason upon the coincident portions of our three terms, and upon these portions only.*

Now let us examine the argument :—

John is taller than William,
William is taller than Charles,
∴ John is taller than Charles.



The representation of this argument is annexed. We here see that what our attention is called to in the first proposition, is the excess of the height of John above that of William, or in the figure to the portion *y z*. The second proposition, in like manner, points to the excess of William's height above the height of Charles, in other words to the portion *p q*. The conclusion refers to the difference between the heights of John and Charles, or to the portion *x y*. Thus, in this kind of argument, *we reason upon the non-coincident portions of our three terms, and upon these only.*

It will be well to observe this figure with care, in order to note a peculiarity of this form of argument. The line *y z* represents the excess of John's height above that of William; the line *x y* its excess above that of Charles. Now *x y* is obviously greater than *y z*, whence it follows that the conclusion does not state in full all that the premisses entitle us to infer. For th premisses are capable of giving us the conclusion—

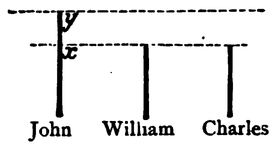
> "John's height exceeds that of Charles more than it does that of William;"

whence we see that the conclusion as we originally had it follows from premisses unnecessarily strong, or stronger than is needed for its proof, whence this kind of argument is called an argument **A Fortiori**. For the same conclusion would have followed from the premisses—

John is taller than William,

William is the same height as Charles,

∴ John is taller than Charles;



and here no stronger conclusion could be substituted.

The argument *a fortiori* is exceedingly useful in geometrical reasoning. Its general form is as follows:—

A is greater than B,
C is greater than A,
∴ C is greater than B.

We have already seen (p. 32) that by the Law of Contradictories, either a proposition or its contradictory must always be true. Thus, if it be not true that All M is P, it must certainly be true that Some M is not P. Now, cases may arise in which we want to prove a certain proposition, but can find no evidence convenient for the purpose. We may desire to establish the truth of "All S is P," and find ourselves unable to discover the relations of S and P to any middle term M. In such a case, we may avail ourselves of the Law of Contradiction, and prove that "All S is P" by means of proving that it is false that "Some S is not P." How, then, can we prove that a proposition is false? Only by proving that something which necessarily fol-

lows from it is false. And this, again, can only be proved to be false by proving that some proposition which necessarily follows from it is false. This process may be continued *ad infinitum*, or we may at last arrive at some proposition, which we know at once, on independent grounds, to be impossible. Then the falsity of our starting proposition, and by consequence, the truth of its contradictory, are at once made manifest.

This form of argument may be represented in symbols, thus—

> It is required to prove that all A is B.
> If 'All A is B' is not true then—
>   Some A is not B.
> But if some A is not B,
> Then [*e.g.*] All C is D;
> And if All C is D,
> Then [*e.g.*] No E is F;
> And if No E is F,
> Then [*e.g.*] Twice two make five.
> But Twice two do not make five,
> Therefore it is not true that No E is F.
> Therefore it is not true that all C is D.
> Therefore it is not true that Some A is not B,
> Therefore All A is B.

It does not in the least matter for our present purpose how we come by the knowledge of the falsity of our final deduction. The fact with which we are concerned is this, that whenever the final conclusion of a piece of reasoning is known by any means to be false, then one of two things must have happened—either we have made some error in the process of our reasoning, or

we have started from a false premiss. On arriving, therefore, at a conclusion known to be false, we may, if we have the slightest doubt of our accuracy, re-examine the process of reasoning by which we were led to the false conclusion, until no doubt remains of the accuracy of that process. Since the process is found to be accurate, nothing remains to account for our false conclusion but the falsity of our original assumption. And since, if that assumption is false, its contradictory must be true, we thus establish the truth of that contradictory.

When the truth of a proposition is thus established, by showing that to assume its contradictory as true leads us to a conclusion which is known to be false, the proposition whose truth is demonstrated is said to be proved *indirectly*, or by **Reductio ad absurdum.**

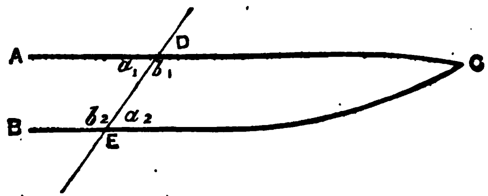
Propositions in mathematics are very frequently established by this method of reducing their contradictories to an absurdity. One such proposition we will examine from a logical point of view :—

> "If a straight line falling on two other straight lines make the alternate angles equal to one another, the two straight lines shall be parallel to one another."—*Euclid*, I. 27; *Wilson's Geometry*, I. 21.

Now, this proposition says that, granted certain conditions, two straight lines are parallel.

"The two straight lines are parallel" has for contradictory "the two straight lines will meet if produced." One of these two contradictories must be true.

Let us take two straight lines and fulfil the conditions, thus :—



Here we have two straight lines, A and B, and another straight line cutting them; and it makes the angle a_1 equal to the angle a_2, and the angle b_1 equal to the angle b_2. It is asserted that, consequently,

A and B are parallel. . . . . *Proposition.*

If they are not parallel, then

A and B will meet if produced. . *Contradictory.*

Suppose this latter to be true; let them be produced to meet in C, and examine what follows.

It follows that D E C is a triangle, of which the exterior angle a_1 is greater than the interior and opposite angle a_2 (Euc. I. 16). And as a_1 was given as equal to a_2, it follows that

Of two equal magnitudes, one is greater than the other. . . . *Absurdity.*

Here we have a conclusion which we know to be false. Either, then, we have made some mistake in our reasoning process, or we started from a false assumption. A rigorous re-examination of our reason-

ing reveals no flaw. Then the *contradictory* assumed as true must have been false; and since the *contradictory* was false, the original *proposition* was true, and it is therefore true that A and B are parallel.

The student cannot be too strongly recommended to examine for himself every case of *Reductio ad absurdum* he meets with in his mathematical studies, until he finds no difficulty in reducing an argument of this kind, however complex, to its skeleton form, as in the instance above.

This method of the *Reductio ad absurdum* will frequently be found in combination with other methods, particularly with the Proof by Exhaustion. The full form of the argument is then as follows :—

> A is either B or C or D or E or . . . .
> Suppose A is B.
> Then it follows that P is Q;
> But we know independently that P is not Q.
> Therefore A is not B;
> Therefore A is either C or D or E or . . . .
> Suppose A to be D;
> Then it follows that R is S;
> But we know independently that R is not S,
> Therefore A is not D;
> Therefore A is either C or E, or . . . .

Similarly we may dispose of all the alternatives but one, which one is thus left established.

This process of *Reductio ad absurdum*, or, as it may also be called, of **Correction of the Premiss,** is of the very greatest importance. It is the most

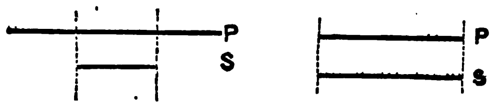
prominent of all the methods by which men learn those truths of Nature that are unitedly known by the name of **Science.** It is impossible for any man, be he never so rude and uncultured, to dwell in the world without noticing in some way the sequences of phenomena by which he is surrounded. Things *are* which *were not*, and which *will not be.* The questions whence these come, whither they go, and why they occur, ever force themselves on man's attention, even in the twilight of a dawning civilization; and for such questions he is sure to frame for himself the best answer within his reach. And such an answer is called a **Theory.** It is, of course, certain that theories so framed will at first be imperfect; but their imperfection can only be discovered by applying them, and by noticing that the conclusions which follow from them do not correspond with the facts. As soon as this is found to be the case, the theory is overthrown. But it is overthrown only to be replaced by another more in accordance with the facts of Nature—that is, more truly *scientific.* In this way knowledge grows.

CHAPTER XI.

OF DEFINITION AND ITS RELATION TO GEOMETRY.

We have already found that the proposition, "All S is P," may be composed of terms either of equal or of

unequal extent. We may represent such a proposition graphically in either of the two following ways:—

(a) 

(b)

In the first of these figures, S forms only part of P; in the second, S constitutes the whole of P. We know also that from the general form, "All S is P," we are not entitled to infer that "All P is S," but only that "Some P is S." But if we have it given us to start with that "All S is P" in the sense represented by figure (*b*) above, we should then be able to convert at once to "All P is S." Such a proposition would really find its full expression in the form, "All S is all P." And wherever we have a proposition of this form—"All S is all P"—where the subject and predicate, though not identical, are of exactly equal extent, and which, consequently, although a universal affirmative, is capable of simple conversion, that proposition, when used to explain the meaning of the term which is its subject, may be called a **Definition.*** To take an old example—

All men are rational animals.

Here not only is the predicate true of the whole of the subject—there being no men that are not rational

* "Let us then consider definition as any conception which, from having precisely the same sphere as another conception, may be used to ascertain its nature and mark out its limits."—ARCHBISHOP THOMSON, *Laws of Thought*, p. 118.

animals—but also the subject is true of the whole of the predicate, there being no rational animals that are not men. The above proposition is, therefore, simply convertible, and a definition of the term *men*.

Definitions are used for the purpose of assigning a meaning to a term. It is a golden rule of method that no term be used in any scientific investigation to which we have not previously assigned a clear and definite meaning, which shall be adhered to throughout that investigation. It would be in vain, for instance, for a man to try to discover whether monarchy was a desirable form of government unless he had previously acquired a clear idea of what he means by the word *monarchy*. In rather more technical language, we may say a man who undertakes to investigate the results of monarchy as a form of government must first "define" a monarch, and must adhere to that definition throughout his investigation. To this end, in order that our definitions may supply us with the clearest possible ideas of the meaning of the words we use, they should conform to the following rules:—

(1.) *The definition must recount the attributes essential to the explanation of the word defined.* To this end we must avail ourselves of the rule we have already investigated (p. 20), that

Genus + Differentia = Species.

By giving for any term its proximate genus, and the difference by which it is distinguished from other species in the same genus, we have at once a definition of that term. Thus, taking the term "Triangle,"

as used in the first book of Euclid, we can arrive at its definition thus :—We have for its proximate genus "Plane Figure," and the triangle differs from all other plane figures in being "contained by three straight lines." Uniting together the genus and the difference, we get, "A triangle is a plane figure contained by three straight lines," a definition which expresses exactly the whole of those attributes which must be present before we apply the word "triangle" to a given object.

(2.) *A definition must not repeat the term defined.* The violation of this rule would obviously give us a definition which did not add in any way to our knowledge of the meaning of the word defined. If I say that a vegetable is that which is possessed of vegetable properties, I clearly leave my hearers exactly where they were so far as their knowledge of the nature of vegetables is concerned. This is a fault known to the old logicians by the Latin name, *Idem per idem* (the same by the same). It is one to which the English language is peculiarly liable, because its composite origin has endowed it with such a wealth of synonyms, and a synonym is often given instead of a definition.

(3.) *A definition must always be simply convertible.* To this end it must express neither more nor less than the precise meaning of the word defined; or, in other words, the predicate must contain exactly the whole of the connotation of the subject. "All men are animals" is not a definition, because the predicate *animals* does not contain the whole of the connotation of *men;* the proposition is not simply convertible into "all

animals are men." But "all men are rational animals" is so convertible, and is therefore a definition.

(4) *A definition must be expressed in the clearest possible terms, without obscure or figurative language.* If this rule be violated, we are defining *ignotum per ignotius*, the unknown by the still more unknown. Pascal thus defines space—"C'est une sphère infinie dont le centre est partout, la circonférence nulle part."* It is needless to say that such a definition does not in the smallest degree help our ignorance of what space is.

It has been well pointed out that in framing a definition intended for the use and information of other persons, we should be careful to make our words not only *clear*, but clear *to them*. "To many a humble thinker, 'honesty is the best policy,' would convey an idea, not adequate indeed, but still distinct, when 'honesty is uprightness in respect to transactions connected with property,' would be but a string of confused words." †

(5) *The definition must not be negative if it can possibly be affirmative.* There are, however, many cases in which the definition cannot be affirmative, for the very term we are defining may be negative in its nature. ‡ "A blind man is one who cannot see," is a case in which a definition is unavoidably negative.

* "An infinite sphere, whose centre is everywhere, circumference nowhere."

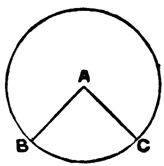
† Archbishop Thomson, "Laws of Thought," p. 118.

‡ Such a term is technically called a *Privative Term.*

It must not be supposed that every term is capable of definition. It is always a difficult matter to obtain a really good definition; and in some cases it is not merely difficult, but impossible, to obtain any definition at all. The names, for instance, of all our simple sensations are terms of this kind. Such words as *hot*, *cold*, *white*, *black*, have no definition in the proper sense of the word; they are *ultimate notions*, and can only be explained by a direct appeal to personal experience.

Definitions, then, are used to explain the meanings of words, to assign the sense in which certain words are to be understood in the course of any investigation we have in hand. Of this nature are the definitions of geometry. When, for instance, we define a circle at the outset of our geometrical studies, we really only mean that hereafter every time the word circle occurs, it shall be understood in the sense now once for all assigned to it. If by the word *circle* we were at one time to mean one kind of figure, and at another another, it would be plainly impossible for anyone to follow reasoning so complex as that of geometry. By adhering to the one meaning throughout, we are able to give to our geometrical reasonings precision for ourselves, and clearness for others. But it must not be supposed that the reasonings of geometry *depend*, in a logical sense, upon the definitions. It cannot be too strongly impressed upon the student's mind that a definition declares the meaning of a word and nothing else. From such a proposition nothing can follow but a conclusion about the meaning of words. Thus

many persons would say that in the annexed figure it "follows from the definition of a circle," that the line A B is equal to the line A C. But the equality of these two lines really follows, not from the definition, but from the *postulate*, hiddenly involved in the definition, that a circle, *i.e.*, a figure with equal radii, can be drawn, or can be supposed to be drawn. If any one thinks that a conclusion as to matters of fact can



be drawn from a definition, he has only to see into what an absurd position he will be driven by trying to draw such a conclusion from the definition of some term which has no real existence, and whose definition, consequently, can contain no such hidden postulate.* Take for instance the term "Dragon." We can define :—

A dragon is a serpent breathing flame.

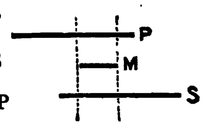
Now by taking this definition to pieces we can get out of it the two propositions :—

A dragon is a thing breathing flame.
A dragon is a serpent.

* The remainder of this investigation is adapted with free alterations from Mill's "Logic," Bk. I., chapter viii.

Making these to stand as the premisses of a syllogism, we shall find that we can give it completely thus:—

A dragon is a thing breathing flame	All M is P
A dragon is a serpent	All M is S
∴ Some serpents breathe flame	∴ Some S is P



As this argument is not in the first figure, we have not examined the special rules which justify it; but we append its symbolical representations, from which we can see that, apart from general rules, and judged on its own merits, this argument is valid. And yet its conclusion is false. This then is a case of *Reductio ad absurdum* examined in the last chapter:—a good process of reasoning lands us in a false conclusion. In such a case our premisses must contain a falsity, and in this instance the falsity will be found to lie in the ambiguous meaning of the copula. (Cf. Chap. V., p. 22). The conclusion asserts that some serpents really exist which do breathe flame, and this assertion only follows from the premisses when the copula in those premisses is understood to imply real existence. But if we alter the premisses to the only form which yields our conclusion,

A dragon is a *really existent* thing breathing flame,
A dragon is a *really existent* serpent,

they at once become false, and not only so, but they cease to be the constituent parts of our definition. Thus the conclusion which we apparently derived from a definition was not drawn from the definition at all, but from a certain assumption which the definition

appeared to warrant, that the thing defined had a real existence. From a definition, therefore, we can draw no conclusion as to the nature of things.

If we wish to find what conclusion we can derive from a definition, we have only to supply the exact force of the copula in the definition, thus—

> Dragon is a word meaning a thing which breathes flame,
> Dragon is a word meaning a serpent,
> ∴Some words meaning serpent, mean also things which breathe flame.

And here we have the only kind of inference which can ever be drawn from a definition as such, namely, an inference as to the meaning of a word.

The advanced student should read on this question Mr. Mill's chapters on Definition, and on Demonstration, Book I., chapter viii., and Book II., chapter v., of his "Logic."

CHAPTER XII.

THE IMPERFECT FIGURES OF THE SYLLOGISM.

In order that the student may not be perplexed by meeting with syllogistic arguments in other figures than the first, and to give some insight into the treatment of such figures when met with, we will now proceed to examine their nature and leading properties.

The method of their formation was explained on page 59. The Second Figure has its elements so

arranged that the middle term is predicate in both premisses. In the Third Figure the middle term is subject in both of the premisses. The Fourth Figure reverses the arrangement of the First Figure, and its middle term is predicate of the major premiss and subject of the minor. We thus have four forms, or *schemes*, the moulds, as it were, into which our syllogistic reasonings may be poured, as follows :—

I.		II.		III.		IV.	
M	P	P	M	M	P	P	M
S	M	S	M	M	S	M	S
S	P	S	P	S	P	S	P

Here we have skeleton propositions of which the two above the line are premisses, the one below the line being the conclusion. The skeletons may be made into the completed organisms of thought by supplying them with the appropriate signs of quantity and quality :—

"All . . . is . . . ," "No . . . is . . . ," "Some . . . is . . ." "Some . . . is not . . . ." And as we saw (p. 60) in the case of the first figure, we shall thus obtain for each of the imperfect figures sixteen possible, but not sixteen *valid*, moods. To distinguish the valid from the possible, the rules of the syllogism must be applied to each mood, just as they were applied to the moods of the first figure on pp. 70—73. This it is absolutely essential that the student should do for himself. The examination of the first figure, as referred to above, will serve as a model, and every mood of the other figures should be similarly examined, properly represented by lines, and either accepted as

a valid argument, or rejected as a fallacy, the fallacy in each case being properly named.

The process when completed will give the following for the valid moods of the various figures :—

Fig. I. AAA, AII, EAE, EIO. [AAI, EAO.]
,, II. AEE, AOO, EAE, EIO. [AEO, EAO.
,, III. AAI, AII, EAO, EIO, IAI, OAO.
,, IV. AAI, AEE, EAO, EIO, IAI. [AEO.]

In the above list the moods enclosed in brackets are in all cases *weakened moods;* those for the first figure having been examined on page 74. They are of small importance, and we will not further mention them. That there are five of them in all shows that there are five, and *only* five moods which can have a universal conclusion. These universal conclusions are negative (E) in every figure but the first. Thus the first is the only figure wherein we can prove a universal affirmative conclusion, and the only one in which conclusions can be drawn of any desired quantity and quality, A, E, I, or O. It is also the only figure to which the *Dictum de Omni et Nullo* (cf. p. 12) will *directly* apply. Hence it was called by Aristotle the *perfect* figure; the second and third figures being by him considered *imperfect.* The fourth figure Aristotle never knew. It is sometimes called the Galenian Figure, from a tradition, of slender authority, attributing its invention to the celebrated Galen; and whether or not its position as a separate figure should be maintained is a question which has been very much disputed.

From an examination of the list of valid moods for each figure as given above, we may discover whether

there are any special rules to which each of those figures must be subject. We have already found them for the first figure (cf. pp. 69, 70) thus:—

(α) The minor premiss must be affirmative.

(β) The major premiss must be universal.

Now in fig. II. the middle term is predicate in both premisses. Hence, since an affirmative proposition leaves its predicate undistributed, we must, in order to distribute our middle term at least once (Rule I., p. 62), have one negative premiss. Hence we have for the Second Figure—

Rule (α).—*One premiss must be negative.*

From this rule it follows necessarily (Rule IV., p. 66) that

Rule (β).—*The conclusion must be negative.*

Now as the conclusion is negative, it distributes its predicate. Hence its predicate, which is the major term, and contained in the major premiss, must in that premiss be distributed. In the second figure the major term is the subject of its premiss: and to distribute the subject of a proposition that proposition must be universal. Hence—

Rule (γ).—*The major premiss must be universal.*

For the Third Figure the scheme is—

$$\begin{array}{cc} M & P \\ M & S \\ \hline S & P \end{array}$$

Now if the minor premiss in this scheme were to be a negative proposition, the conclusion also would have to be negative (Rule IV.) But the negative conclusion would distribute P, which must then be

distributed in the premisses also. But this would require *two* negative premisses, which is impossible. Wherefore—

Rule (α).—*The minor premiss must be affirmative.*

Again, if the conclusion were an A, then S would have to be distributed in the premisses as well as in the conclusion, which could only be done by making the minor premiss negative. But if either premiss were a negative, the conclusion could not be an affirmative. Wherefore an A is an impossible conclusion in this figure. For similar reasons so also is an E. Hence—

Rule (β).—*The conclusion must be particular.*

Lastly for the Fourth Figure. Its scheme is—

$$\begin{array}{cc} P & M \\ M & S \\ \hline S & P \end{array}$$

If the major premiss here is affirmative, it leaves its predicate, the middle term, undistributed. And as the middle term must be distributed once (Rule I.), the minor premiss must be in such a case universal. Hence—

Rule (α).—*When the major premiss is affirmative, the minor must be universal.*

Again, if the minor premiss is affirmative, S is undistributed in the premisses, and must therefore be undistributed in the conclusion. Hence—

Rule (β).—*When the minor premiss is affirmative, the conclusion must be particular.*

Again, if the conclusion be negative, it distributes P, which must therefore be distributed in the premisses.

This can only be done by making the major premiss universal. Hence—

Rule (γ).—*In negative moods the major premiss must be universal.*

Again, if the conclusion were an A, it would distribute its subject S. But S could only be distributed in the premisses if the minor premiss were negative, thus involving a negative conclusion. Moreover, if the major premiss were a particular negative, P would be undistributed in the premisses and distributed in the conclusion, which would have to be negative, thus giving illicit process of the major term; whilst if the minor premiss were a particular negative, the middle term would not be distributed in the premisses at all. Hence we have—

Rule (δ).—*The conclusion cannot be a universal affirmative, nor either premiss a particular negative.*

We have thus found what are the valid moods of each figure, and the rules which in each figure regulate and determine their validity. Now it is necessary to *know* what moods are valid in each figure without having always to investigate them one by one. Looking again at their enumeration as given above, the task may well seem hopeless to fix them firmly in the memory. This may, however, be readily done by means of a device, as follows:—The vowels have consonants added to them to form words, and the moods we have enumerated are collected together into the following mnemonic hexameter lines, of which the first was given on page 73:—

Barbara, Celarent, Darii, Ferioque prioris:
Cesare, Camestres, Festino, Baroko secundæ:
Tertia Darapti, Disamis, Datisi, Felapton,
Bokardo Ferison, habet: quarta insuper addit
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

These lines must be learnt by heart. As will soon appear, not only every word, but almost every letter they contain, is of importance. They were for the most part the invention of Petrus Hispanus, who became Pope John XXII., and died in 1277. Sir William Hamilton justly says of them that "there are few human inventions which display a higher ingenuity"; and Professor De Morgan describes them as "the magic words by which the different moods have been denoted for many centuries,—words which I take to be more full of meaning than any that ever were made."

We have seen already that to the imperfect figures the *Dictum et Omni et Nullo* will not directly apply. The ancient logicians considered it of the highest importance to be able to apply that dictum to every argument. Therefore when an argument presented itself to them in the form of one of the imperfect figures, they endeavoured to find means for throwing that argument into the form of the first figure. And the mnemonic lines given above convey full directions for this process of changing an argument of the second, third, or fourth figure into an equivalent one of the first figure,—a process known to logicians by the name of **Reduction.**

Definition.—A mood of the second, third, or fourth figure is said to be reduced to the first figure when precisely the same conclusion is obtained in the first

figure from premisses equivalent to and derived from the premisses of the original syllogism.

Now Reduction may be of two kinds:—

A. **Ostensive.**

B. **Per Impossibile** (*i.e.*, *Reductio per deductionem ad impossibile*).

These are distinguished from each other thus:— Ostensive Reduction uses only the ordinary processes of immediate inference, with or without transposition of the original premisses. Reduction *per impossibile* shows by means of the first figure that the contradictory of the conclusion of the given argument is false, and, therefore, that the conclusion in question is itself true. These processes are indicated by the consonants in the mnemonic lines, thus:—

The *initial* letter in the name of each mood will show to which of the moods of the first figure that mood must be reduced. Thus Festino must be reduced to Ferio, Disamis to Darii, Camenes to Celarent, etc. Other consonants in each word will tell what process must be employed for the reduction. Thus—

s	stands for	simple conversion;
p	„	conversion *per accidens*;
m	„	transposition of premisses (*mutando*);
k	„	reduction *per impossibile*.

The other letters are non-significant; and the letters *s* and *p* apply to the proposition indicated by the vowel *preceding* them. The letter *k* only occurs twice, in Baroko and Bokardo. This is due to the fact that the old logicians never employed the method *per impossibile* if they could avoid it; and they did not know how to reduce these two moods ostensively. It

will be seen, however, that we are now able to reduce *any* mood either ostensively or by the *per impossibile* method. We shall proceed to give some examples of Reduction.

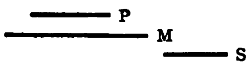
I. Reduce *Camestres* to the first figure ostensively. *Camestres*, Figure II., must be reduced to *Celarent.*

Scheme of Figure II. is—

P M
S M
∴ S P

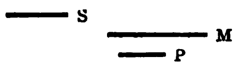
Camestres is then—

All P is M,
No S is M,
∴ No S is P.



Following the letters we have by the first *s* and by the *m* that the minor premiss must be converted simply, and then the premisses transposed. This gives—

No M is S,
All P is M,
∴ No P is S.



Celarent, valid.

This is a valid conclusion in *Celarent*, in Figure I. But the conclusion is not the *same*, but is the converse of the original conclusion. The final *s* in *Camestres* solves this difficulty, and converting simply our new conclusion, we have—

No S is P.

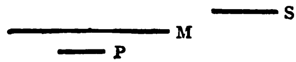
The student should practise the methods of reduction given in this chapter until they become perfectly

familiar. The process should be written down as concisely as is consistent with the clear indication of every step; thus—

Camestres to *Celarent.*

Camestres is—

All P is M,
No S is M,
∴ No S is P.



By the letters we have—

No M is S,
All P is M,
∴ No P is S;
∴ No S is P.

Celarent, valid.

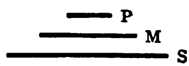
On no account should the student permit himself to neglect the testing both of the given and of the reduced mood by the method of lines. Let him aim at saving time, not by the omission of the test, but by obtaining such a facility in its use that its application shall not represent more than three rapid dashes of the pen, and an almost imperceptible delay.

We will take one more example of ostensive reduction:—

Bramantip to *Barbara.*

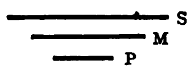
Bramantip is—

All P is M,
All M is S,
∴ Some S is P.



By the letters we have—

All M is S,
All P is M
∴ All P is S;
∴ Some S is P.



Barbara, valid.

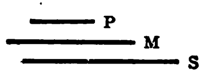
In similar fashion we can reduce all the moods, though there are two, *Baroko* and *Bokardo*, the ostensive reduction of which will require further explanation. But there is another way of establishing, by means of the first figure, the conclusions whether of these two or of any other imperfect moods; and this method was the only one known by the ancient logicians to be applicable to these two. This is the "*Reductio per Impossibile.*"

We have already seen that the truth of a proposition can be established by showing that to assume its contradictory as true, leads to a conclusion that we know to be false. The process of thus indirectly establishing a truth was called *Reductio ad absurdum* (cf. pp. 88-92). Now the process of *Reductio per impossibile* is really a case of this: we prove in the first figure that the assumption of the truth of the contradictory of the conclusion of our given mood would involve a new conclusion which we know to be false, and consequently that that contradictory must itself be false—*i.e.*, that the original conclusion must be true.

For example—Reduce *Baroko*, *per impossibile.*

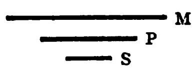
Baroko is—

All P is M,
Some S is not M,
.·. Some S is not P.



If this conclusion is not true, its contradictory is true. Assume such contradictory for a new *minor* premiss. We shall have—

All P is M,
All S is P,
.·. All S is M.



Barbara, valid.

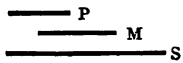
But this conclusion contradicts the original minor premiss, which was given as true. This new conclusion must therefore be false; therefore one of the premisses from which it is derived must be false. Now "All P is M" cannot be false, since it was given as true at first; then "All S is P" must be false, therefore its contradictory must be true, therefore it is true that—

Some S is not P.

Similarly to reduce *Bokardo, per impossibile.*

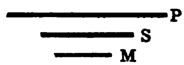
Bokardo is

Some M is not P,
All M is S,
∴ Some S is not P.



If this conclusion is not true, its contradictory is true. Assume such contradictory for a new *major* premiss. We shall have—

All S is P,
All M is S,
∴ All M is P.



Barbara, valid.

But this conclusion contradicts the original major premiss, which was given as true. This new conclusion must therefore be false; therefore one of the premisses from which it is derived must be false. Now "All M is S" cannot be false, since it was given as true at first; then "All S is P" must be false; therefore its contradictory must be true; therefore it is true that—

Some S is not P.

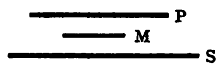
In the last two examples it should be carefully noticed that in the second figure it is the *minor*, and in the third figure, the *major*, premiss which gives place to the contradictory of the conclusion.

This indirect method is applicable to *any* of the imperfect moods, notwithstanding that the mnemonic lines only give instructions for its use in two of the moods. For example—

Reduce *Darapti*, *per impossibile.*

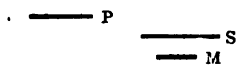
Darapti is—

All M is P,
All M is S,
.·. Some S is P.



If this conclusion is not true, its contradictory must be. Assume such contradictory for new *major* premiss. We have—

No S is P,
All M is S,
.·. No M is P.



But this conclusion contradicts the original major premiss, wherefore, etc.

Further, just as reduction *per impossibile* is applicable to every mood, so every mood can be ostensively reduced by the aid of processes of immediate inference. Thus, to reduce *Baroko* ostensively—

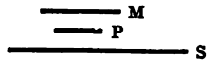
For *Baroko* read *Fagovo*,

and let *g* indicate Contraposition, and *v* Obversion (cf. pp. 43-47). We then have—

Fagovo to *Ferio.*

Fagovo is—

All P is M,
Some S is not M,
∴ Some S is not P.



Following the letters, we have—

No not-M is P,
Some S is not-M,
∴ Some S is not P.

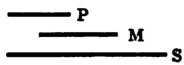
erio, valid.

Similarly we may reduce *Bokardo* ostensively.

For *Bokardo* read *Dogamog*, and reduce *Dogamog* to *Darii.*

Dogamog is—

Some M is not P,
All M is S,
∴ Some S is not P.



Following the letters, we have (N.B., transpose premisses)—

All M is S,
Some not-P is M,
∴ Some not-P is S,
∴ Some S is not P.

Darii, valid.

The student will now be enabled to treat any argument in any figure as the arguments of the first figure have been treated. Practice and experience are the only sources of skill and readiness in the handling of arguments. But there is no better practice for those who would be accurate themselves

(which is of much importance) or skilful in detecting inaccuracy in others (which is of less importance) than is to be found in manipulating arguments in strict logical form.

CHAPTER XIII.

IRREGULAR AND COMPOUND SYLLOGISMS.

We have hitherto treated of syllogisms all the elements of which were explicitly stated. And, moreover, each syllogism we examined stood alone and was judged on its own merits, without any reference to the fact that in any prolonged piece of reasoning many syllogisms will occur closely blended together into a complex whole. We will now proceed to consider some of the forms which the syllogism is wont to assume when abbreviated or compounded in obedience to the practical necessities of reasoning.

Even here, however, the student will observe that there will be some difference between arguments as they appear on the logician's dissecting table and as they move about in the haunts of men, working to convince or to persuade. This difference is unavoidable. For Logic and Rhetoric are ever closely associated, and the argument whose dead weight alone it is the logician's function to measure appears in the senate or on the platform instinct with the life and clothed with the form and beauty with which it is the mission of eloquence to endow it.

Considering first the syllogisms not all of whose elements are explicity stated, we may find that some completely omit to state in words one of their three propositions. This is the most common form in which syllogistic reasoning is met with in practice. For most of the major premisses in common use consist of well-known general truths, or of general propositions which rightly or wrongly pass for general truths amongst a large number of persons. Thus we may hear it said, "The Member for X is a Conservative, so he voted against the Reform Bill." Here we have an argument which omits the major premiss necessary to its validity, viz., "All Conservatives voted against the Reform Bill." But though the major premiss is the part of the argument most commonly suppressed, it is not so always. We may also hear it said, "All the Conservatives voted against the Reform Bill, so the Member for X is sure to have done so." Here is omitted the minor premiss, "The Member for X is a Conservative." And we may also find that the premisses are fully given, but the hearer is left to draw his own conclusion, as when we are told, "All the Conservatives voted against the Reform Bill, and the Member for X is a Conservative." Any such incompletely expressed syllogism is called an **Enthymeme**, and we have—

Definition. An Enthymeme is a syllogism abbreviated in its expression by the omission of one of its three propositions. If it omits the *Major Premiss* the Enthymeme is of the *First Order;* if the *Minor Premiss*, it is of the *Second Order;* if the *Conclusion*, it is of the *Third Order.*

The name Enthymeme is derived from the Aristotelian 'Ενθύμημα, which term is itself compounded from ἐν and θυμός, "in the mind." But it would be a great error to suppose that the modern enthymeme is the same as the Aristotelian ἐνθύμημα. The modern enthymeme is called by Aristotle *συλλογισμὸς μονολήμματος*, "a syllogism with only one lemma." He would not have allowed that such a syllogism is "imperfect," because he considered arguments to be perfect or imperfect, *οὐ πρὸς τὸν ἔξω λόγον, ἀλλα πρὸς τὸν ἐν τῇ ψυχῇ λόγον*, "not according to their external form of expression, but according to the form of reasoning the mind." The Aristotelian enthymeme, however he did call "imperfect," because it was imperfect in its process. It was only a *probable* argument, convincing to the arguer's own mind (ἐν θυμῷ), but not fitted for convincing an opponent. It was an argument ἐξ εἰκότων ἢ σημείων, "from likelihoods and signs," whereof the εἰκὸς states a general likelihood, *e.g.*, "Most landsmen who go to sea are seasick," and the σημεῖον gives a fact as a sign that the likelihood is in operation, *e.g.*, "A certain landsman sailed for America yesterday." The conclusion, "That landsman is seasick to-day," is one the landsman would probably be prepared to admit; though the argument is imperfect, as his friends ashore may still entertain hopes of its falsity. Thus Aristotle called a syllogism which was *imperfect as an instrument of conviction* an enthymeme. Modern logicians, finding that the Aristotelian enthymeme was an "imperfect argument," applied the name to an argument *imperfect in expression*, a wholly different kind of imperfection.

It should be carefully noticed that enthymemes are frequently the vehicles of material fallacies, and especially afford opportunity for the particular fallacy of *Petitio Principii*, or Begging the Question. A sophistical* reasoner may make use of an enthymeme of the first order so as to serve two purposes at once. That is, he may assert the expressed conclusion in the particular case in question, to which conclusion there may be no objection, and he may imply the truth of the general principle contained in the unexpressed major premiss, which may not be by any means equally sound. Thus perhaps you may meet Mr. Brown's medical adviser and express to him a hope that Mr. Brown, who you hear has been attacked with small-pox, will not die of it; and he may reply, "Oh no, he will not die of it, he has been vaccinated." Two purposes are here served. That Mr. Brown will not die of small-pox is *asserted*, and this may be true; whilst the impression of the *implied* major premiss, "No vaccinated people die of small-pox," is left on your mind without the speaker having burdened his conscience with expressly saying so. Thus in examining any enthymematic argument the first attention should always be given to the suppressed portion of it.

Having thus treated of arguments in which there

* A sophism resembles a fallacy in that both are invalid arguments. But the fallacy deceives the reasoner himself and may or may not deceive his hearers; whilst the sophism does not deceive the arguer himself and is more or less deliberatel intended to deceive his hearers.

is less than one completely expressed syllogism, we may examine some which contain more than one syllogism. For when we have proved a certain conclusion by means of certain premisses, we may then desire to prove those premisses themselves. A syllogism constructed for this purpose is called as regards the original syllogism a **Prosyllogism,** the original syllogism being called with respect to the prosyllogism an **Episyllogism.** If we combine many syllogisms in accordance with this plan, any one of them except the first and the last may be regarded as either prosyllogism or episyllogism. The first will always be a prosyllogism and the last always an episyllogism. And such a combination of syllogisms is a **Polysyllogism** or **Train of Reasoning.**

But it will not be necessary to express fully any of the syllogisms which compose the train. We may use an enthymeme to prove either the major or the minor premiss of our original syllogism, or both; and we thus obtain an

Epicheirema. When an enthymeme stands in the place of either premiss we have a single epicheirema; if of both, a double epicheirema.

Thus to take a symbolic example of the double epicheirema:—

All M is P for it is X,
All S is M for it is Y,
∴ All S is P.

We have here in the place of the two premisses two enthymematic syllogisms of the first order in the

mood Barbara, as we can show by decomposing the epicheirema thus :—

I.	All X is P,	II.	All Y is M
	All M is X,		All S is Y,
$\therefore$	All M is P.	$\therefore$	All S is M.

And having thus supplied the missing elements of our enthymematic premisses the decomposition is completed by taking our conclusions, and from them drawing the original conclusion—

III. All M is P,
All S is M,
$\therefore$ All S is P.

It may sometimes be necessary to establish by syllogistic methods a logical relationship between two classes, which though really related as containing and contained, are yet separated by a very wide range of denotation and connotation. Thus Londoners are certainly bipeds, but between these classes there are many intermediate classes, or *subalterna genera.* A form of argument can be constructed which shall trace this relationship through as many as may be desired of the intermediate classes, in either direction ; and from the necessarily prolonged and as it were "piled up" character of this argument it has been called Sorites, from the Greek σωρός, a heap. The sorites has generally been very loosely defined. The following definition is that given by Ueberweg :—

Sorites is an episyllogistic train of reasoning whose expression is simplified by the omission of all the conclusions save the last, and whose expressed conclusions

are identical with the major or minor premisses of the following syllogisms.

Should the definition appear somewhat formidable that appearance will cease on comparing it with the two forms of an actual case of the argument. Taking our two classes as before, Londoners and Bipeds, we may reason thus—

I. All Londoners are Englishmen,
All Englishmen are Europeans,
All Europeans are Men,
All Men are Bipeds,
∴ All Londoners are Bipeds.

Or thus—

II. All Men are Bipeds,
All Europeans are Men,
All Englishmen are Europeans,
All Londoners are Englishmen,
∴ All Londoners are Bipeds.

Of these two forms the first only was, perhaps, known to Aristotle, and it is therefore called the Aristotelian sorites; the second form is called the Goclenian, being attributed to Goclenius. In symbols they stand thus—

I. Aristotelian.	II. Goclenian.
All A is B,	All D is E,
All B is C,	All C is D,
All C is D,	All B is C,
All D is E,	All A is B,
∴ All A is E.	∴ All A is E.

There can of course be any number of premisses in a sorites. And we shall find that a sorites of n premisses will always consist of $n-1$ syllogisms, into which syllogisms it can be resolved. Thus to resolve the examples given above :—By noticing the marks * and †,

the student will see that in the Aristotelian sorites the suppressed conclusion of the one syllogism becomes in each case the *minor* premiss of the next. In the Goclenian form the conclusion of the one syllogism becomes the *major* premiss of the next.

I.	ARISTOTELIAN.	II.	GOCLENIAN.
α.	All B is C,	α.	All D is E,
	All A is B,		All C is D,
	∴ All A is C; *		∴ All C is E; *
β.	All C is D,	β.	All C is E, *
	All A is C, *		All B is C;
	∴ All A is D; †		∴ All B is E; †
γ.	All D is E,	γ.	All B is E, †
	All A is D, †		All A is B,
	∴ All A is E.		∴ All A is E.

A sorites may aim to establish either an affirmative or a negative conclusion. But any syllogism which has one negative premiss must have a negative conclusion, and as that negative conclusion will, in a sorites, form one of the premisses of the next syllogism, it is clear that to have more than one negative premiss in a sorites would sooner or later lead to our having a syllogism in that sorites with two negative premisses, which would invalidate the whole sorites. And again, since a particular premiss gives a particular conclusion, and since we cannot have two particular premisses in a syllogism, it is similarly clear that we cannot have more than one particular premiss in a sorites. Moreover, looking again at our examples, since the conclusion of our sorites, if negative, will distribute its predicate, that predicate must be distributed in the premisses. Therefore the one possible negative pre-

miss must be the one which has the predicate of the conclusion for its predicate, that is, the last premiss in the Aristotelian and the first in the Goclenian form. This fixes the position of the one possible negative premiss.

Where then may the one possible particular premiss be? In the Aristotelian sorites the suppressed conclusion becomes in each case the *minor* premiss of the next syllogism. Therefore every premiss in such a sorites, except the first, will in turn become a *major* premiss. Now the *major* premiss in Fig. I. must always be universal (Rule β, p. 70). Hence in the Aristotelian sorites the one possible particular premiss must be the first. In the Goclenian sorites, on the other hand, the suppressed conclusion becomes in each case the *major* premiss of the succeeding syllogism, and therefore must be universal. Neither of the premisses from which it is derived can thus be particular, and the only place the one possible particular premiss can occupy is that of minor in the last syllogism of the sorites. That is, it must be the last premiss in a Goclenian sorites. Hence we have the special rules of the sorites as follows :—

Rule α.—Only one premiss can be negative; and if one is negative it must be the last in the Aristotelian dan the first in the Goclenian sorites.

Rule β.—Only one premiss can be particular, and if one is particular it must be the first in the Aristotelian and the last in the Goclenian sorites.

It should be noticed that it is doubtful whether the word sorites as used here was known to Aristotle at all. The oldest use of the term was to designate a

peculiar sophism which consisted in asking, "Do three barleycorns make a heap?" "Do four?" "Do five?" and so on till at last the opponent was obliged to change his answer from "No" to "Yes," with the apparently absurd result of admitting that a single barleycorn made the difference between that which was and that which was not a heap.

APPENDIX

ON THE WORDS "AXIOM" AND "POSTULATE."

To examine, or even to state, all the questions that cluster round these two words would be quite beyond the range of this work. At the same time, they hold so important a position in geometrical reasoning, that they cannot be passed without mention.

There can be no question that there are in the human mind certain beliefs of very wide generality of application, and of a truth which, independently of any special investigation, we all hold to be absolutely unfailing. Such truths we call **Axioms.** Whence come these beliefs, and whether or no they be capable of any kind of demonstration, are questions which, together with many others concerning them, the student will find discussed to his heart's content in larger works on Logic. It is sufficient for our present purpose to say that for all ordinary reasonings, we are quite satisfied with the evidence in favour of any proposition when we have shown that it follows logically from one of these axiomatic truths.

The most common use of the axiom in geometrical reasoning, as well as the most effective, occurs when it is made the final result in the process of *Reductio ad absurdum*. Thus in proving Euclid I. 6, we argue that certain sides of a triangle are equal to

each other, for if not, then the part is equal to the whole; but the whole is greater than its part by an axiom, therefore the sides are not unequal, that is, they are equal.

The **Postulates** are simply requests to be allowed to perform certain operations necessary to the reasonings we undertake, and without which we cannot proceed to argue. If you will not allow me to describe a circle whenever I desire, then I cannot make one line of the same length as another, and the whole fabric of geometry becomes impossible. Thus the postulates are the *condition* of geometrical reasoning, but form no part of that reasoning themselves.

EXAMPLES

THE following collection of examples is intended to be rather suggestive than exhaustive. It must always lie with the practical skill of the teacher to exactly apportion to the needs of the individual pupil the exercises he shall perform.

Some of these examples are original; for some the author is indebted to friends, for some to various examination papers, and for a very few to other works.

One or two will be found not *exactly* to refer to anything in the text. They must be solved by the exercise of the student's common sense, and, in the hands of a skilful teacher, will furnish a valuable incentive to habits of exact thought.

EXAMPLES

1. Assign the following words to the class or classes of mental phenomena to which they relate (Sensation, Emotion, Volition, or Thought):—

Anger, Resolution, Bitter, Smarting, Frightened, Because, Rough, Weak-minded, Cold, Hot, Pretty, Joyful, Suppose, Heavy, Acute, Loving, Blue, Shrill, Fragrant, Despair.

2. What are the parts into which an argument may be divided?

3. May a man be said to reason badly because his conclusions are false?

4. Distinguish between the logical and the grammatical predicate.

5. Define a Syllogism, and state the meaning of the word.

6. Construct a syllogism containing the three terms "Useful," "Hammers," "Tools." Name its parts, and represent it in two ways by methods of notation.

7. What is the Copula, and what is its function? Explain what is meant by the copula not being *explicit*, and show how its function is performed in such a case.

8. Define a Term. Why is it so called?

9. Which of the following words are terms?—Good, Why, Animal, Of, Suppose, Away, Not, He, File, Saw.

10. Distinguish between the denotation and the connotation of a term. Explain as nearly as you can what is the denotation and connotation of the following terms:—School-board, Candlestick, Penholder, Insect, Fossil, Red, Mr Gladstone, Newspaper, Quadruped, Star, Deaf.

11. What difference of denotation and connotation is there between the two terms "Rose" and "Moss-rose?"

12. Arrange in ascending order of connotation and in descending order of denotation the following terms:—Dog, Animal, Spaniel, Quadruped, Vertebrate.

13. Explain fully why the two arrangements in answer to the last question are identical.

14. Find the *logical* sum of—

5 sharks + 3 sprats.	6 roses + 7 lilies.
7 eagles + 2 jackdaws.	7 lilies + 8 ferns.
6 men + 9 women.	8 ferns + 9 dogs.
7 eagles + 3 sprats.	3 circles + 4 parabolas.
6 circles + 4 triangles.	$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$.

Express the result in each case in terms of the smallest possible denotation.

15. Express the following propositions in strict logical form, and append to each the letter designating its quantity and quality:—

The noblest study of mankind is man.
In a war justice can be but on one side.
The rolling stone gathers no moss.
It is never too late to mend.

Extremes meet.
A crocodile is a kind of lizard.
He can't be wrong whose life is in the right.
The circle is one of the conic sections.
He who runs against time has an antagonist that knows no accident.
In folly's cup still laughs the bubble joy.
No song, no supper.
Who lasts a century can have no flaw.
The books men live by are not the books that live.
Non omnia possumus omnes.

All had not paid who entered.
They weave no garlands but to lay on graves.

Then none was for a party;
Then all were for the state;
Then the great man helped the poor,
And the poor man loved the great.

16. Take the proposition, "All sciences are useful," and determine precisely what it affirms, what it denies, and what it leaves doubtful concerning the relations of the terms "Science" and "Useful thing."

17. It being given us that the proposition, "All crystals are cubes," is false, make a list of all the propositions that we thence know to be true.

18. "All equilateral triangles are equiangular." What other propositions follow from this?

19. "Not every table is a piece of furniture, for the multiplication table is not." Criticize this sentence.

20. Show how to get the converse of the contrary of the contradictory of the proposition, "Some crystals are cubes." How is it related to the original proposition?

21. What do you mean by one proposition being compatible with another? Write two propositions logically compatible with "All dogs are animals." Is "Some dogs are animals" compatible with it? If not, why not?

22. Construct as completely as possible a table similar to that on page 50 for each of the other forms of proposition, E, I, O.

23. From a proposition can we be said to *infer* its contrary? If not, what can we *infer* with respect to contraries?

24. Having given as true that "All sharks are fishes," write about sharks and fishes two propositions whose truth or falsity is unknown.

25. Assign the logical relations, if any, between each of the following propositions and the first of them:—

All crystals are solids.
Some solids are not crystals.
Some not-crystals are not solids.
No crystals are not solids.
Some solids are crystals.
Some not-solids are not crystals.
All solids are crystals.

26. How many terms may a syllogism contain, and why?

27. Explain fully the functions of the Middle Term in a syllogism.

28. Explain by aid of a diagram why the Major and Minor Terms are so called.

29. "I am enough of a logician to know that from false premisses it is impossible to draw a true conclusion." Examine this statement.

30. "A line is length without breadth." Are there, then, any lines?

31. Examine the following syllogisms; express them by a method of notation; state whether they are logically correct; and, if fallacious, name, if you can, the fallacy they contain:—

(i.) All metals are opaque,
Gold is a metal,
∴ Gold is opaque.

(ii.) Bodies occupy space,
The moon is a body,
∴ The moon occupies space.

(iii.) Lunar eclipses coincide with the full moon,
Last night we had a lunar eclipse,
∴ Last night was full moon.

(iv.) All men are mortal,
No dogs are men,
∴ No dogs are mortal.

(v.) Some men are fools,
All philosophers are men,
∴ Some philosophers are fools.

(vi.) All fishes live in the water,
Some vertebrates are fishes,
∴ Some vertebrates live in the water.

(vii.) All Swedes have fair hair,
Some Europeans are not Swedes,
∴ Some Europeans have not fair hair.

(viii.) What you buy in the market to-day you will eat to-morrow,
Raw meat is what you buy in the market to-day,
∴ You will eat raw meat to-morrow.

(ix.) Some oysters are not fit to eat,
This is a heap of oysters,
∴ Some in the heap are not fit to eat.

(x.) Some oysters are fit to eat,
No eggs are oysters,
∴ Some eggs are not fit to eat.

(xi.) All bread is food,
No poison is bread,
∴ No poison is food.

(xii.) Six and seven are even and odd,
Thirteen is six and seven,
∴ Thirteen is even and odd.

(xiii.) All pigs are birds,
All apples are pigs,
∴ All apples are birds.

(xiv.) All pigs are birds,
No apples are pigs,
∴ No apples are birds.

(xv.) No pigs are birds,
No apples are pigs,
∴ No apples are birds.

(xvi.) Wise men are just,
This man is foolish,
∴ He is unjust.

(xvii.) He who steals a biography takes a man's life; therefore he deserves hanging.

(xviii.) A portion of the Londoners is Conservative; now, as this man is a Londoner, he is partly a Conservative.

(xix.) I always thought he was a respectable man—he kept a gig.

(xx.) Adieu to virtue if you're once a slave.

32. Construct syllogisms in each of the four moods—Barbara, Celarent, Darii, Ferio.

33. The mood with premisses E A is valid, the mood with premisses A E is not. Explain the cause of the difference.*

34. The middle term must be distributed once; can it be distributed twice? Prove your answer.*

35. If the major term is distributed in the premisses, what do we know about the conclusion? Prove your answer.

36. If the major term is undistributed in the premisses, what do we know about the conclusion? Prove your answer.

37. State and explain clearly the rule about the Middle Term of a syllogism.

38. Obvert the proposition, "There is no truth in the statement that the ship has been lost."

39. Write the contrary and contradictory of the proposition—

"He knows to live who keeps the middle state."

* It must be remembered that these questions apply here only to the Logic of the First Figure.

40. What do you mean by *Differentia?* How is it related to Genus and Species?

41. In Barbara and Celarent respectively, what do we know as to the truth of the conclusion in each of the following cases?—

i. Major premiss false, minor premiss true;
ii. Major premiss true, minor premiss false;
iii. Both premisses false.

Give the fullest explanation you can of your results.

42. Find results similar to those of the last question for Darii and Ferio.

43. State and prove the law of the relation between the truth of the antecedent and that of the consequent in a hypothetical argument.

44. Distinguish between a Constructive and a Destructive Hypothetical Syllogism.

45. State the conditions of the validity of the *Modus Ponendo Tollens.*

46. Examine the following arguments; reduce them if you can to the categorical form; and explain their faults, if any:—

(i.) If there is a rainbow, the sun is shining;
The sun shines;
∴ There is a rainbow.

(ii.) If there is a rainbow, the sun is shining;
There is a rainbow;
∴ The sun shines.

(iii.) If there is a rainbow, the sun is shining;
The sun is not shining;
∴ There is no rainbow.

(iv.) If there is a rainbow, the sun is shining;
There is no rainbow;
∴ The sun is not shining.

(v.) To be successful, a man must be either very clever or very fortunate; this successful man has been very fortunate; therefore he is not very clever.

(vi.) There are two trains (early morning and late evening) to the station I want to go to. I do not like early rising, so I must take the night train.

47. What conclusions, if any, follow from the subjoined premisses? (*N.B.*—Express all the arguments in full logical form.)

(i.) This man is either a knave or a fool; and he seems to have his wits about him.

(ii.) One of us two is lying, and I am telling the truth.

(iii.) One of us two must rule, and the other obey; and I mean to rule.

(iv.) To get into Parliament, a man must have either great wealth or great talents; and this man is neither wealthy nor talented.

(v.) To get into Parliament, a man must have either great wealth or great talents; and the member for X is a poor man.

(vi.) To get into Parliament, a man must have either great wealth or great talents; and the member for X is very rich.

48. Distinguish between the argument *à fortiori* and the ordinary syllogism.

49. What do you mean by a *Theory?* Analyse the process of *Reductio ad absurdum*, and show the part it plays in the growth of scientific theories.

50. What is a Definition? Define, if possible, the terms in Question 10, pointing out the exceptions it may be necessary to make to any of the rules of definition. How far do the reasonings of geometry depend on the Definitions, on the Postulates, on the Axioms?

51. What name do you give to the argument at the end of Euc. I. 18 (Wilson, I. 10)? Name any other propositions where the same argument is adduced in proof.

52. What faults can you observe in Euclid's definition of parallel lines? Do you consider that the same objections hold with regard to his definition of a point? Can you suggest any improvement?

53. Name the process of argument used in Euc. I. 25 (Wilson, I. 16). Display it fully in symbols, and name any other propositions proved in the same way.

54. Reduce to its skeleton form, and express in symbols, where so reduced, the argument of Euc. I. 26 (Wilson, I. 7 and 17).

55. Do the same for Euc. I. 29 (Wilson, I. 22).

56. Also for Euc. I. 39 (Wilson, II. 4).

57. Also for Euc. I. 40.

58. "The angles at the base of an isosceles triangle are equal to one another." Express this as a hypothetical proposition, and when so expressed, convert it.

59. Are the enunciations of Euclid, in your opinion, hypothetical or categorical propositions? Give reasons for your reply.

60. Taking Euc. I. 32 (Wilson, I. 25), name and describe by symbols, and, if possible, by methods of notation, all the arguments used for its establishment, starting from first principles.